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Risk-constrained strategic bidding of a hydro producer under price uncertainty

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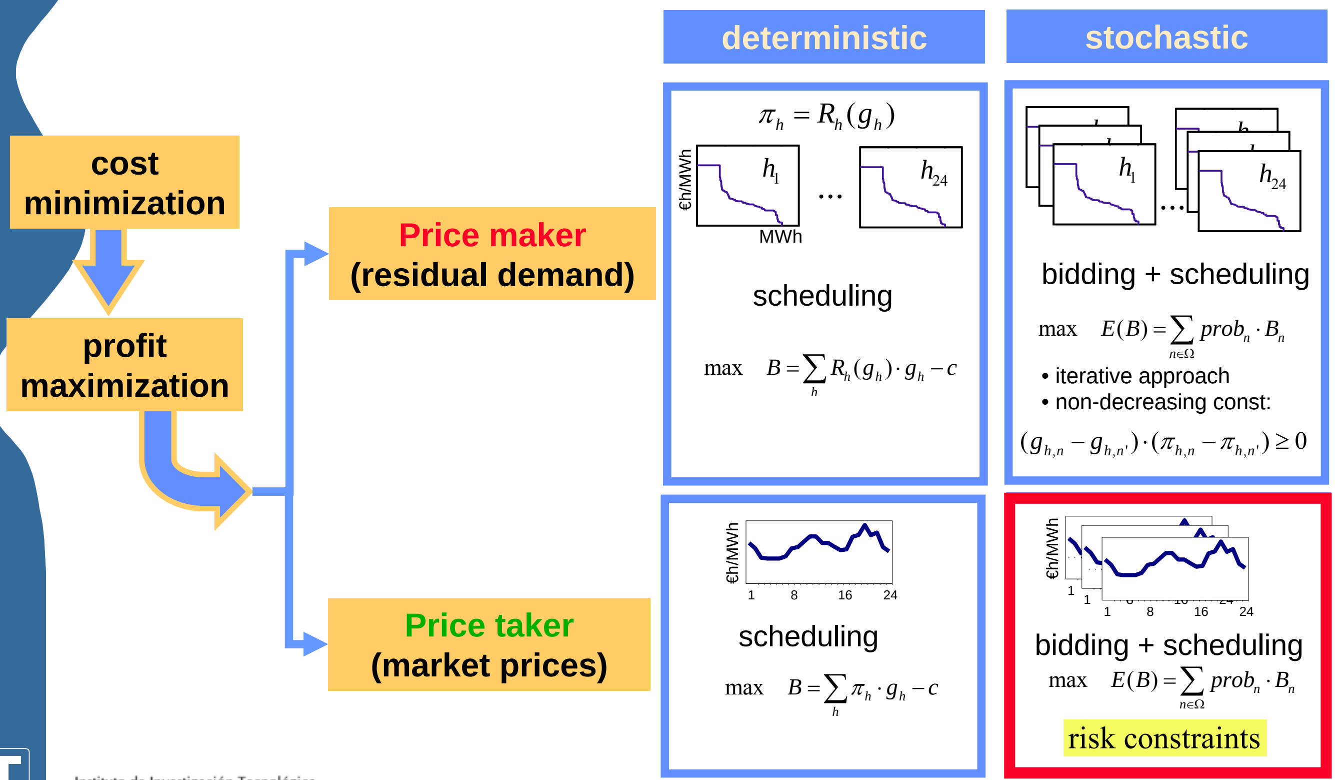
Introduction

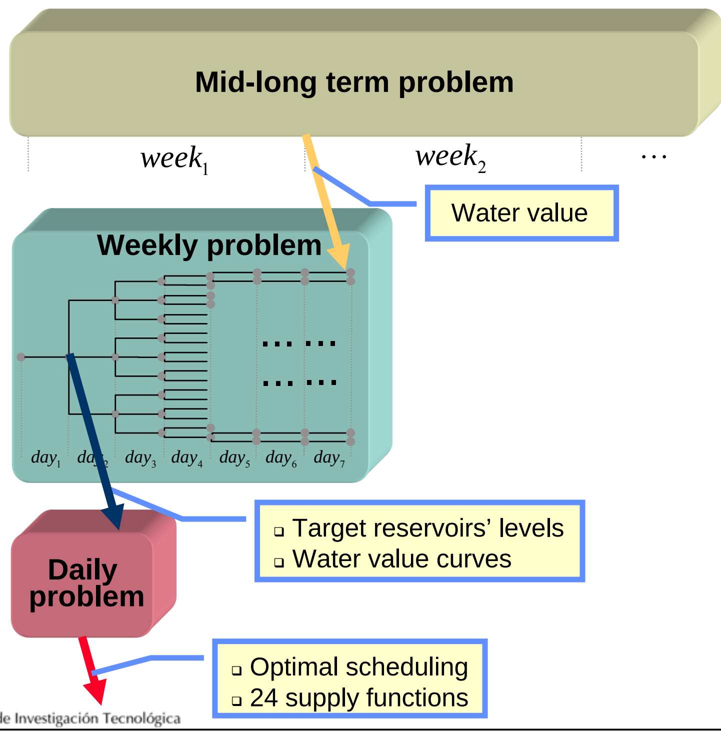
- Research experience
- Hydroelectric utilities need models for:
  - Short-term hydro scheduling in pool-based electricity markets
  - Strategic bidding
- The operational margin of generation companies can be significantly lower than expected under certain realizations of market uncertainty.
- How to incorporate risk aversion into the short term decisions?
- We propose to include the **CVaR (and GCVaR)** as a hedging strategy:
  - Minimum CVaR constraints; CVaR in the objective function

Introduction: background



Hierarchy of hydro models

- The daily problem in the context of a temporal hierarchy



Pereira et al.
Fleten et. al
Nabona et al.
García-González et al.
…

Nürnberg & Römisch, 02
Nowak & Römisch, 00
…

Feltenmark & Lindberg, 97
Conejo et al., 02
Pritchard, 04
García-González et al., 07
…

Mathematical formulation (1)

Max [objective function]

subject to

- Water balance: $$v_{ik} = v_{i(k-1)} + w_{ik} - (q_{ik} + s_{ik}) + \sum_{j \in \Omega_i} (q_{j(k-\tau_{ji})} + s_{j(k-\tau_{ji})})$$
  $$\underline{v}_i \leq v_{ik} \leq \overline{v}_i$$
- Water rights: $$\underline{\theta}_{ik} \leq q_{ik} + s_{ik} \leq \overline{\theta}_{ik}$$
- **Input-output characteristic curve modeling**
- **Water value**
- **Non-decreasing curves constraints (for the strategic bidding problem)**
- **Risk-management constraints**

Risk neutral objective function

- The risk neutral objective function is:

$$\text{Maximize: } \overbrace{\sum_{n=1}^{N} prob_n \cdot B_n}^{\text{expected profit}}$$

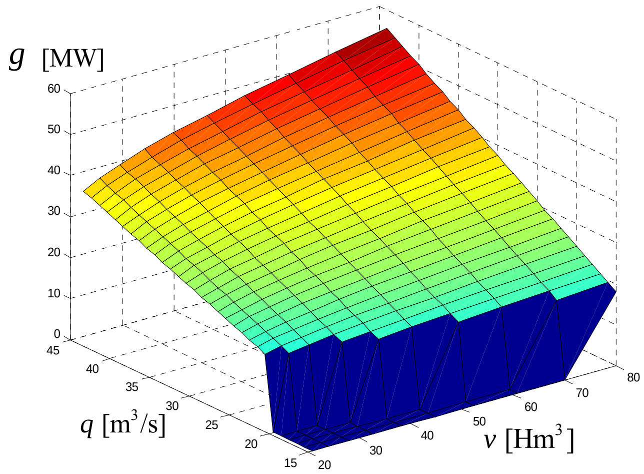
where $prob_n$ is the probability and B_n is the profit in scenario n:

$$B_n = \overbrace{\pi_{kn} \cdot \sum_{i=1}^{I} \left(g_{ik} \right)}^{\text{daily incomes in scenario } n} - \overbrace{\sum_{i=1}^{I} \sum_{k=1}^{K} \left(c_i \cdot y_{ik} \right)}^{\text{start-up costs}}$$

Nilsson & Sjelvgren, 97

Input-output characteristic curve: head dependency

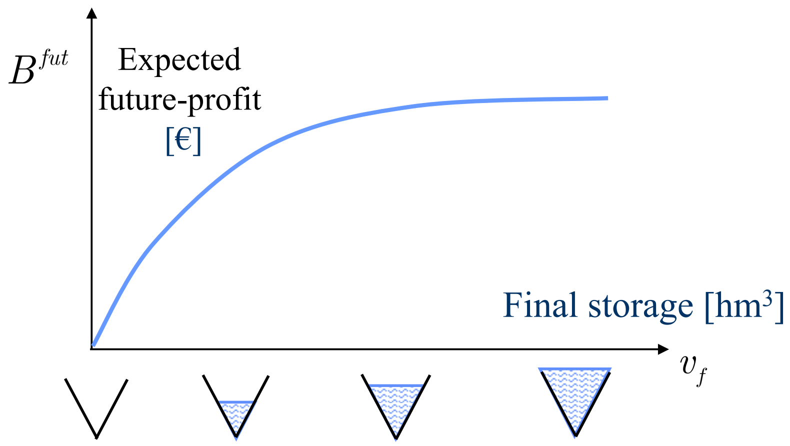
- We have followed several approaches in order to consider the effect of the net head of hydro electric plants:



- To consider a fix head — details
- Meshing and triangulation of the surface using MILP — details
- To consider a family of input-output curves with binary variables — details
- Under-relaxed procedure — details
- Quadratic Programming approach — details

Water value (1/2)

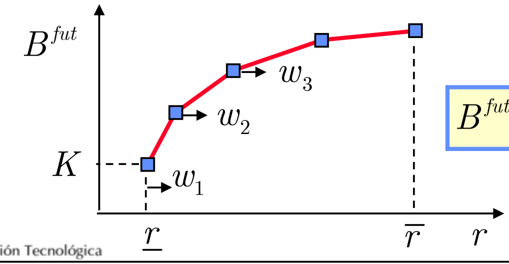
- The expected future-profit depends on the level of the reservoirs at the end of day:



Water value (2/2): several alternatives

- Fixed target levels at the end of the day $v_{i,24} \in [v_i^f - \varepsilon, v_i^f + \varepsilon]$
  - ✔ pros: easy to understand by practitioners
  - ✘ cons: it loses the opportunity of selling more (less) generation in case market prices are high (low)
- Individual water value curves for each reservoir (PMAPS-06)
- Expected future-profit curve depending on the total energy storage in the river basin at the end of the day (IEEE-PES GM-07)

$$r = \sum_i c_i^{avg} \left(v_{i,24} - \underline{v}_i \right) \qquad \underline{r} \le r \le \overline{r} \qquad r = \underline{r} + \sum_s \omega_s$$



$$B^{fut} = K + w_1 \cdot \lambda_1 + w_2 \cdot \lambda_2 + w_3 \cdot \lambda_3 + \ldots$$

Non-decreasing curves constraints

- The heuristic procedure presented in PMAPS-06 to build the bid curves is substituted by a set of constraints in order to ensure the monotonicity of the supply functions.
- Price taker: no product of variables as in the oligopolistic approach
- Portfolio allowed?
  - YES

$$\sum_i g_{n,i,k} \ge \sum_i g_{n',i,k} \ , \forall \left\{ (n,n') \middle| \pi_{nk} \ge \pi_{n'k} \right\}, \forall k$$

  - NO

$$g_{n,i,k} \ge g_{n',i,k}, \forall \left\{ (n,n') \middle| \pi_{nk} \ge \pi_{n'k} \right\}, \forall k, \forall i$$

Risk measures

- A risk measure is a mapping from the random variables representing risks (profits or losses) to the real line that provides a simple number that quantifies the risk exposure and can be used to compare different investment alternatives, financial positions, etc.
- Among others, convex and coherent risk measures are becoming a powerful tool in financial risk management due to their axiomatic foundation and their favorable computational properties.
- Let assume that the risk can be quantified on the basis of a random variable $X : \Omega \mapsto \mathbb{R}$ defined on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$
- For a given scenario, $n \in \Omega$ the realization of the random variable is $X(n)$ that can represent for instance the profit from selling hydroelectric power in the spot market.
- A risk measure can be defined as a mapping $\rho : X \mapsto \mathbb{R}$

Coherent risk measures (Lüthi & Doege, 2005)

- $\rho : X \mapsto \mathbb{R}$ is a "convex risk measure" if it satisfies:

$$\rho(\lambda X + (1-\lambda)Y) \leq \lambda\rho(X) + (1-\lambda)\rho(Y) \qquad \lambda \in [0,1], \forall X, Y \in \mathcal{X}$$

$$X \leq Y \Rightarrow \rho(X) \geq \rho(Y) \qquad X(n) \leq Y(n)\ \forall n \in \Omega$$

$$\rho(X + a) = -a + \rho(X) \qquad a \text{ constant}$$

- If it satisfies also the following, it is called "coherent risk measure"

$$\text{if } \lambda \geq 0 \Rightarrow \rho(\lambda X) = \lambda\rho(X)$$

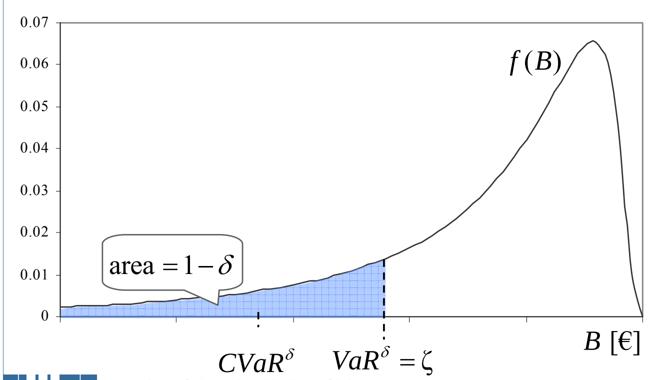
- We will restrict our attention to the CVaR and the GCVaR. Assuming that $(-X-\eta)_+ = \max\{0, (-X-\eta)\}$:

$$CVaR_\beta(X) = \inf_\eta [\eta + \frac{1}{\beta}\mathbf{E}_\mathbf{P}[(-X-\eta)_+]]$$

$$\left. \begin{array}{l} GCVaR_\beta(X) = \inf_\eta [\eta + \frac{1}{\beta}\mathbf{E}_\mathbf{P}[(-X-\eta)_+]] \\ subject\ to \quad \sum_{n\in\Omega} (-X(n)-\eta)_+ \leq L \end{array} \right\}$$

Risk management constraints: CVaR

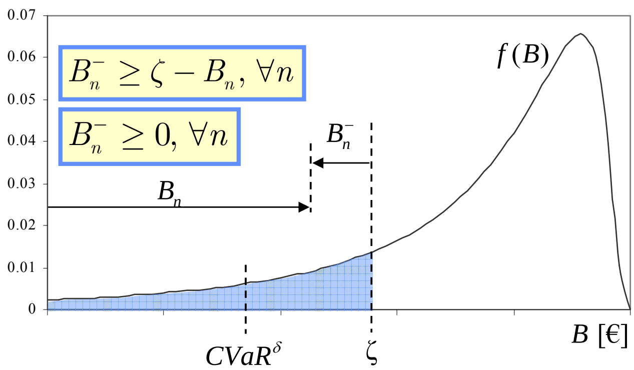
- CVaR (Conditional Value at Risk) was firstly introduced in the financial sector by Rockafellar and Uryasev in 1999. It has been applied subsequently to energy markets (Unger, 02), (Cabero et. al, 05), etc.
- Normally, it is expressed in terms of losses. In this case, it is expressed in terms of profits (it is maximized)
- Assuming that ζ represents the VaR (Value at Risk), then CVaR can be defined as: $CVaR^{\delta}(B) = \mathrm{E}(B \mid B < \zeta)$



$$CVaR^{\delta}(B) = \frac{\sum_{n \in N | B_n < \zeta} prob_n \cdot B_n}{\sum_{n \in N | B_n < \zeta} prob_n} = \frac{\sum_{n \in N | B_n < \zeta} prob_n \cdot B_n}{1-\delta}$$

Risk management constraints: CVaR

For the discrete setting, is can be formulated linearly:



As a constraint: $\mathrm{cvar}^{\delta} \geq CVaR^{\delta}_{\min}$

$$\mathrm{cvar}^{\delta} = \zeta - \frac{\sum_{n \in N} prob_n \cdot B_n^-}{1-\delta}$$

As a term in the objective function

Risk averse objective function

- The risk averse objective function can be expressed as:

$$\text{Maximize:} \qquad \alpha \overbrace{\left(\sum_{n\in\Omega} prob_n \cdot B_n \right)}^{\mathbf{E}(B)} + (1-\alpha)\cdot CVaR$$

GCVaR

$$\left. \begin{array}{l} GCVaR_\beta(X) = \inf_\eta [\eta + \frac{1}{\beta}\mathbf{E}_\mathbf{P}[(-X-\eta)_+]] \\ subject\ to \quad \sum_{n\in\Omega}(-X(n)-\eta)_+ \leq L \end{array} \right\}$$

- Parameter *L* can be used to control the risk tolerance while maintaining a simplified linear computation:

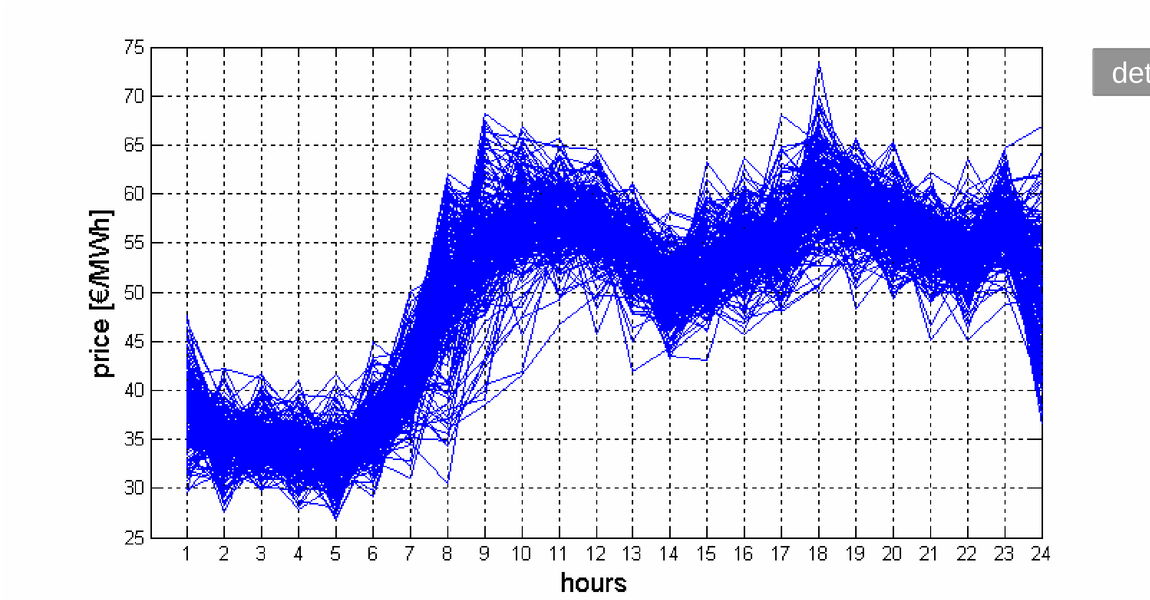
$$L \geq \sum_{n\in\Omega}(-X(n)-\eta_{VaR})_+ \quad \Rightarrow \quad GCVaR_\beta(X) = CVaR_\beta(X)$$

$$L = 0 \quad \Rightarrow \quad GCVaR_\beta(X) = MaxLoss(X)$$

- Therefore, GCVaR encompasses CVaR, although it can be used to penalize small profits in the tail of the distribution function.

Prices scenarios

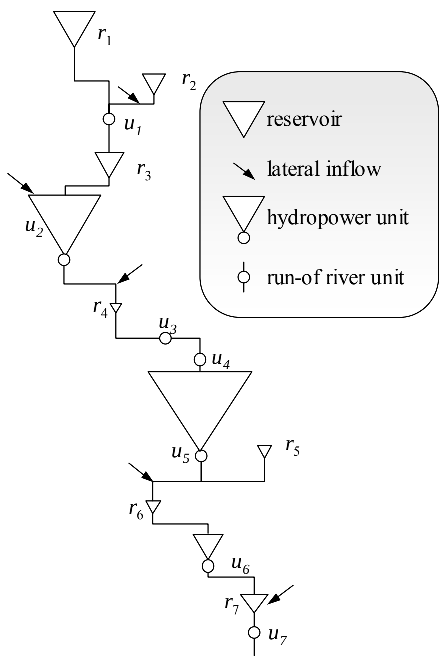
- We have used the Input-Output Hidden Markov Model (IOHMM) to generate price scenarios (n=250)



details

Example case

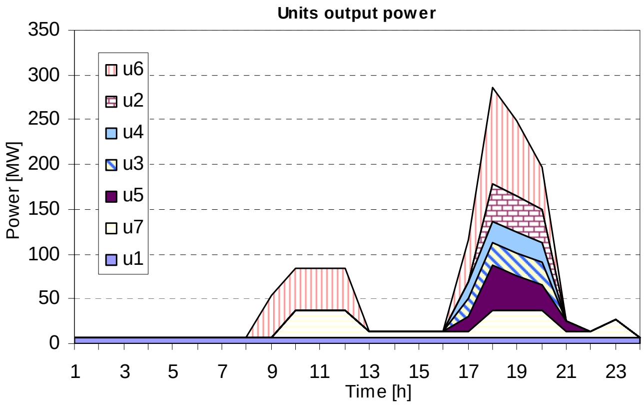
- The model has been tested with a fictitious but realistic case.



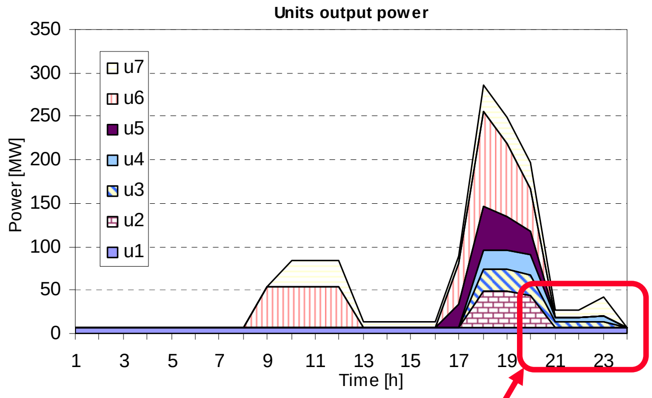
	$\bar{v}$	$\underline{v}$	v^o	v^f	$\bar{p}$	$\bar{q}$	w	c
	[Hm^3]				[MW]	[m^3/s]		[€]
u1	-	-	-	-	12	10	5	30
u2	85	20	45	45	56	45	1	125
u3	-	-	-	-	30	38	-	75
u4	-	-	-	-	24	36	-	67.5
u5	213	80	123.5	122	58	110	-	145
u6	20	13	16.4	16.7	120	124	-	350
u7	-	-	-	-	31	120	-	85
r1	40	15	25	25	-	9	-	-
r2	15	3	12	12	-	11	-	-
r3	23	6	19.4	19.1	-	31	-	-
r4	0.49	0.11	0.3	0.3	-	35	1	-
r5	1.5	0.6	1.38	1.3	-	48	-	-
r6	1.3	0.1	0.72	0.67	-	56.5	1	-
r7	12.55	7.1	10	9.79	-	120	12	-

Scheduling

Risk neutral



Minimum CVaR



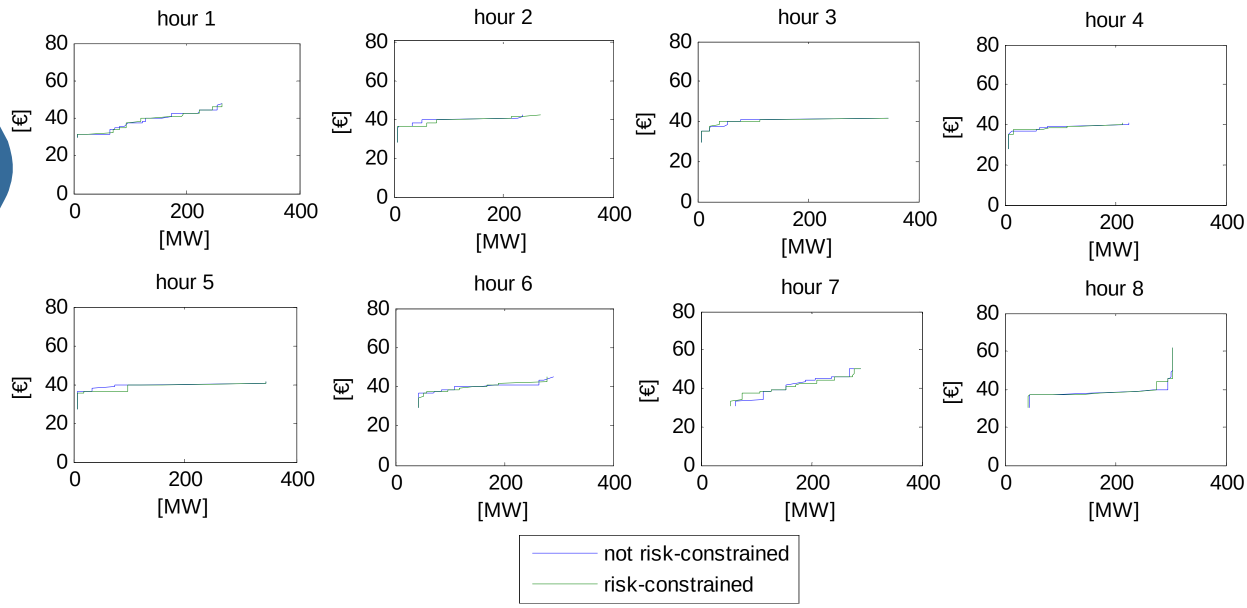
Generation during [h21:h23] is increased by "energy" from peak to off-peak hours.

Strategic bidding

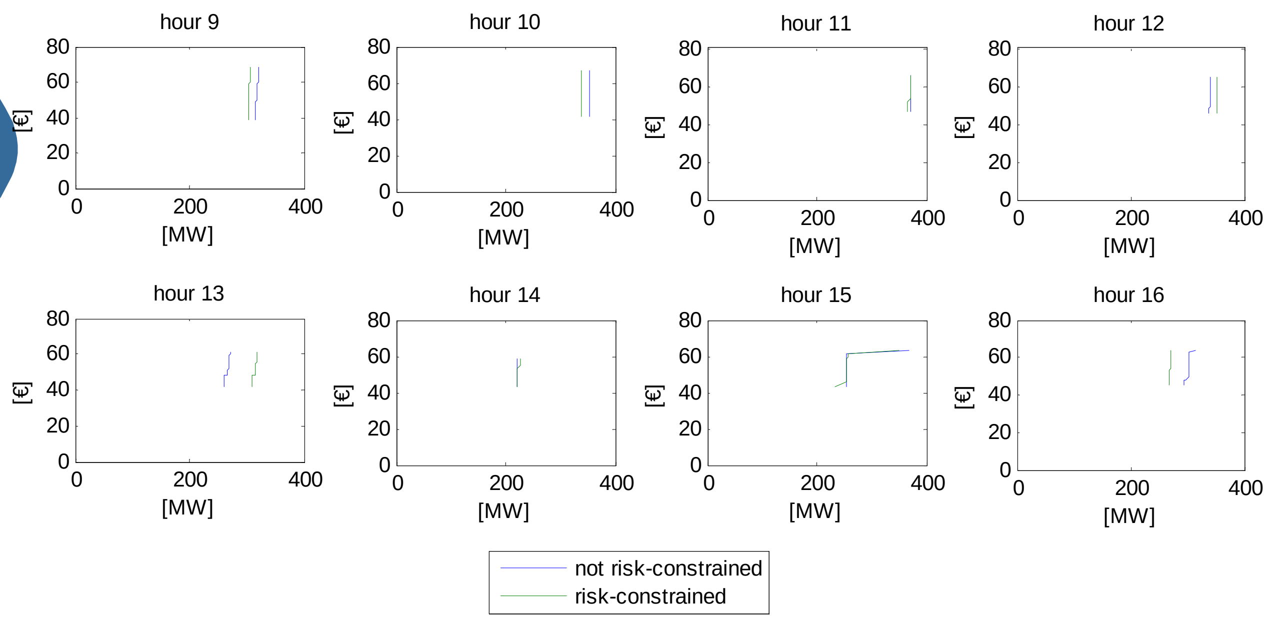
$$\text{Maximize:} \quad \alpha \overbrace{\left(\sum_{n \in \Omega} prob_n \cdot B_n \right)}^{\mathbf{E}(B)} + (1-\alpha) \cdot CVaR$$

Parameter α	VaR [€]	CVaR [€]	Expected Profit [€]
1	547700	543970	555250
0.5	548090	544310	555170
0	548140	544530	547780

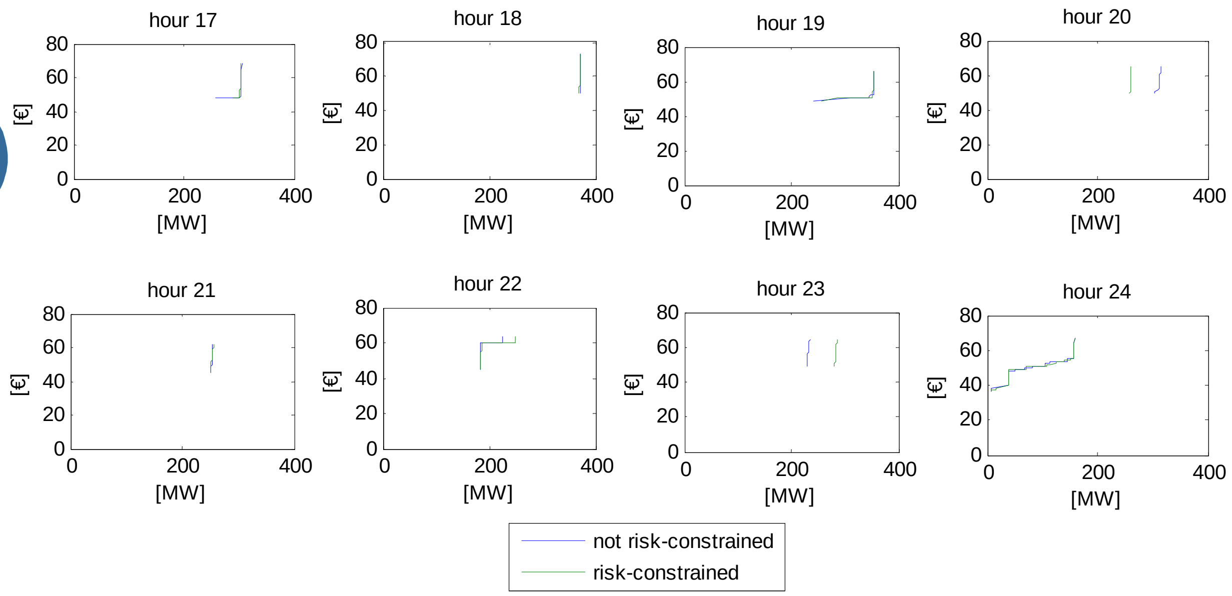
Obtained curves (alpha=1; alpha=0.5) h1-h8



Obtained curves (alpha=1; alpha=0.5) h9-h16



Obtained curves (alpha=1; alpha=0.5) h17-h24



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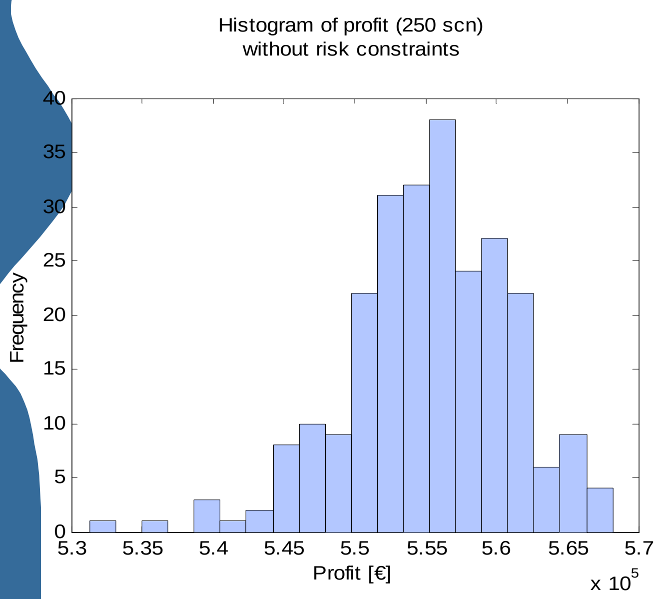
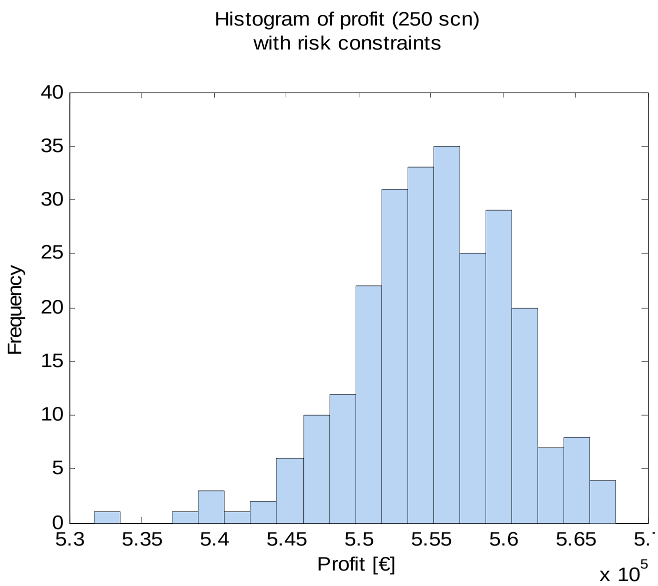
Profit histograms (day + future)





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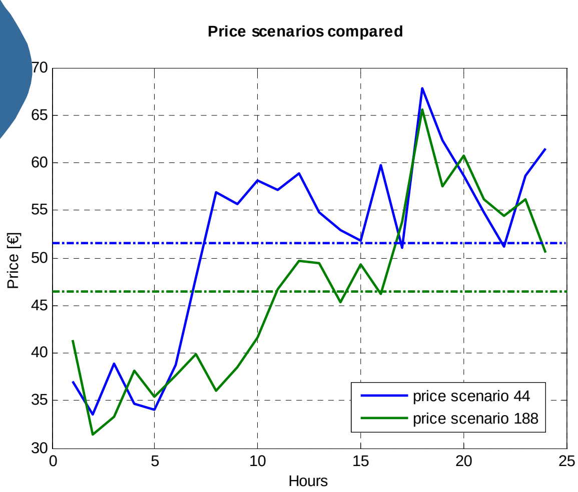
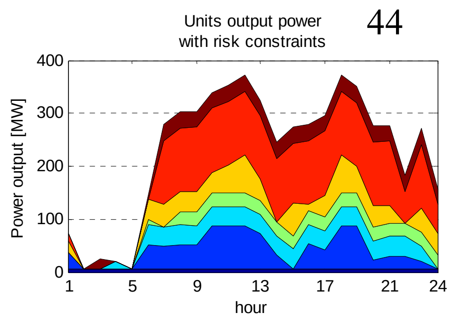
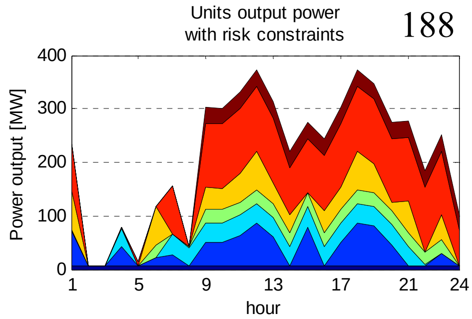
- For each price scenario, a different scheduling is obtained:

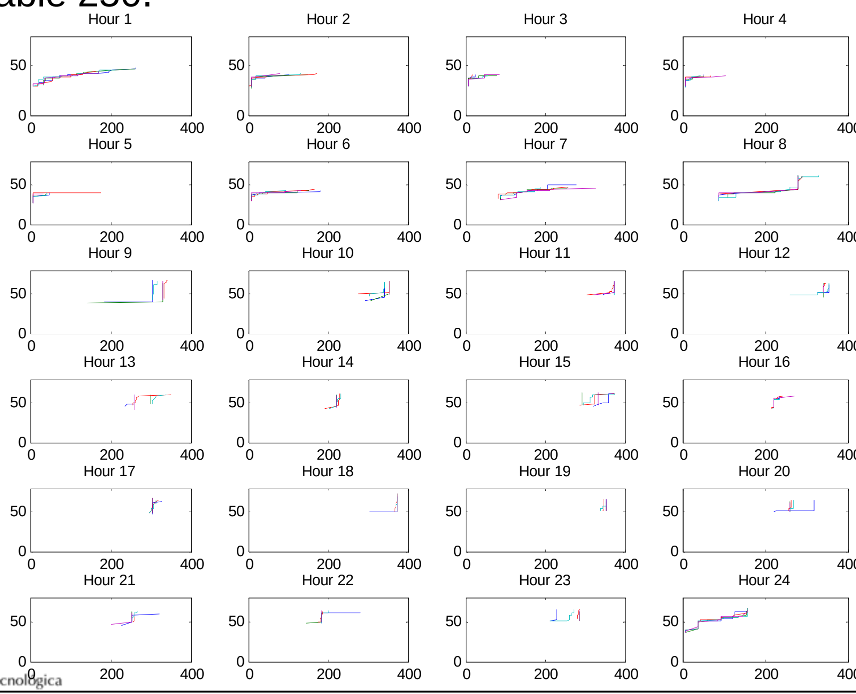




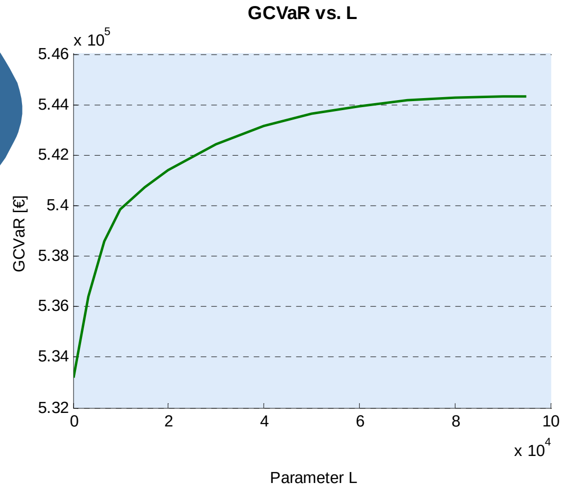


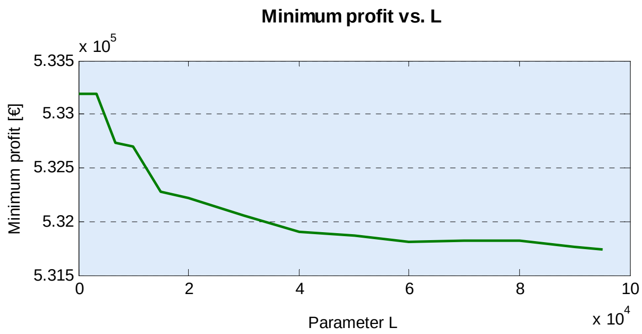
Effect of the number of scenarios?

- The model was run taking 70 random price scenarios from the available 250.



GCVaR study





Conclusions

- We have presented an optimization model to help a price taker hydro-generation company in day-ahead market to:
  - Find the optimal scheduling when fixing the final target levels.
  - To build the generation bids of its hydroelectric units when water value curves a available.
- We have fixed previous drawbacks.
- The CVaR and GCVaR constraint has proven to be an effective risk-aversion criterion as shown in the example case presented in the paper.
- Future developments: the case of a price-maker.

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Thanks for your attention!

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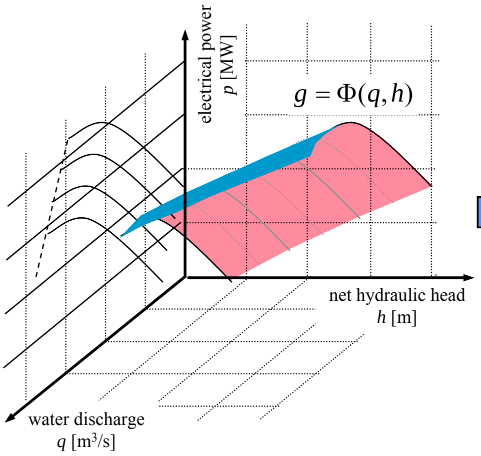
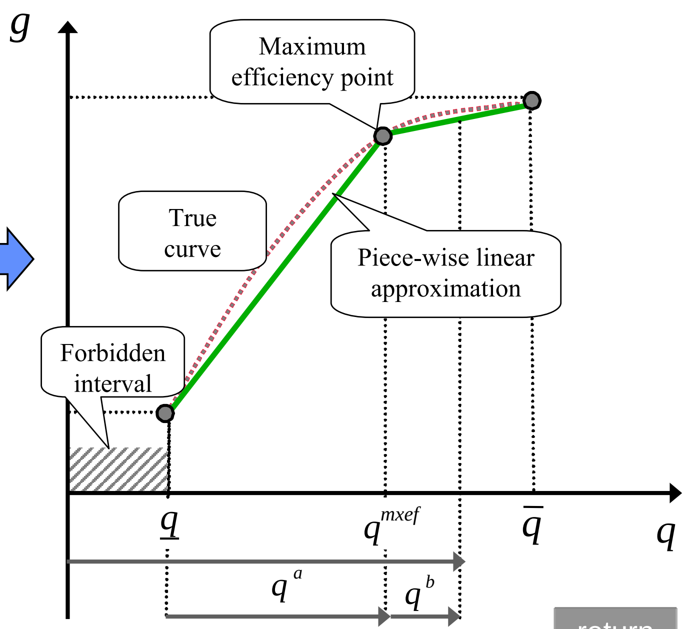
IEEE Power Engineering Society General Meeting, Tampa 2007

Annexes

- Fixed head Input-Output curve
- Meshing and triangulation of the surface using MILP
- Family of input-output curves with binary variables
- Under-relaxed iterative procedure
- Quadratic Programming approach
- IOHMM

Fixed head Input-Output curve





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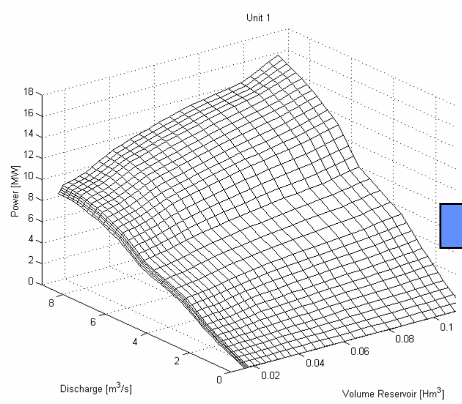
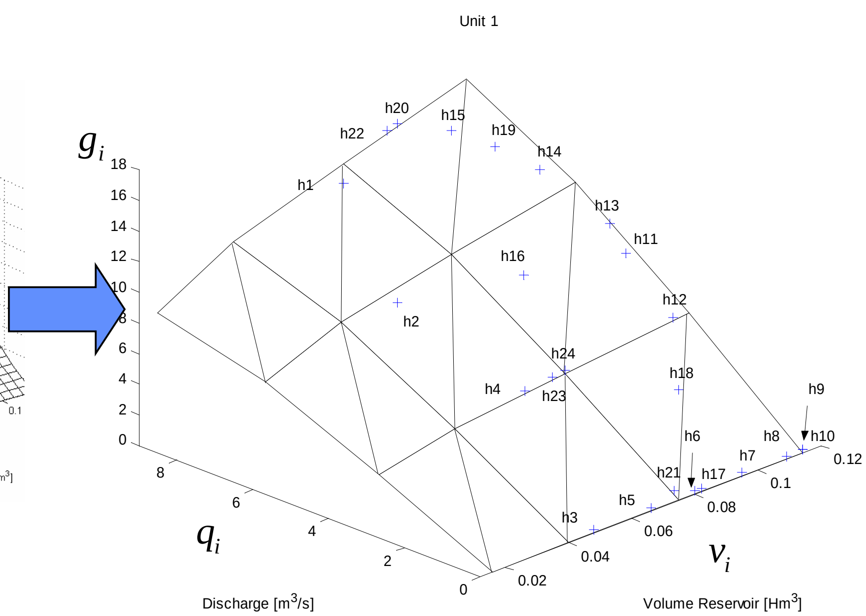
Meshing and triangulation of the surface using MILP





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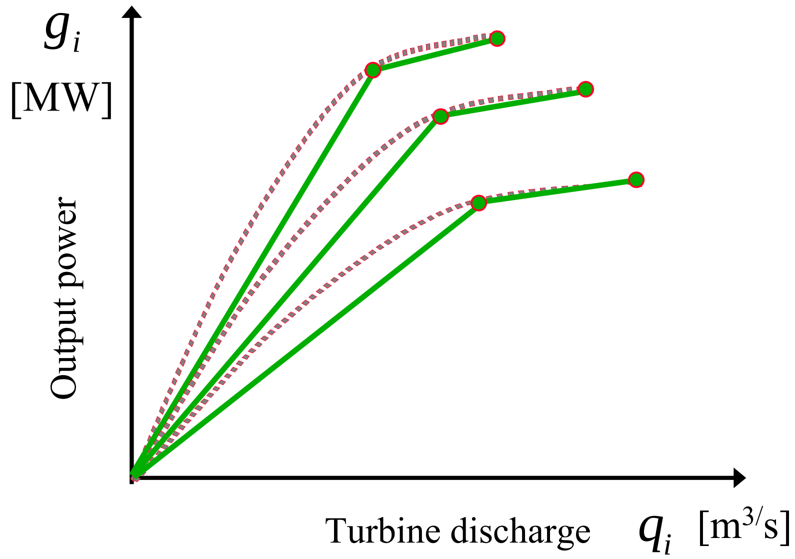
Family of input-output curves with binary variables



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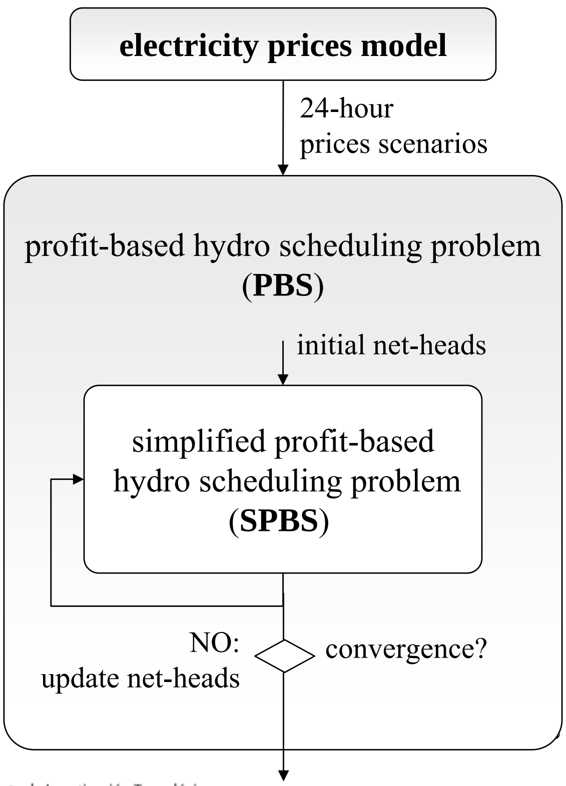
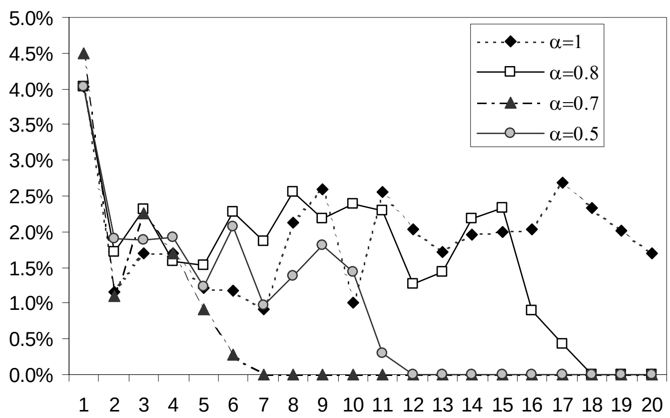
Under-relaxed iterative procedure



$$h_i = \rho_i(v_i)$$

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}\right)$$

$$h_{ik}^{\upsilon+1} = \rho_i\left(v_{ik}^{\upsilon+1}\right) = \rho_i\left(v_{ik}^{\upsilon} + \alpha \cdot \left[v_{ik} - v_{ik}^{\upsilon}\right]\right)$$



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Quadratic Programming approach

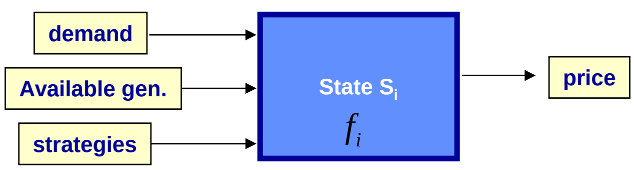
- The characteristic surface can be approximated by a quadratic polynomial: $g = c_1 \cdot v^2 + c_2 \cdot q^2 + c_3 v \cdot q + c_4 \cdot v + c_5 \cdot q + c_6$
- CPLEX is an efficient solver for quadratic programming problems
- Needs to be positive semi-definite $g = c_1 \cdot v^2 + c_2 \cdot q^2 + c_4 \cdot v + c_5 \cdot q + c_6$
- We sample the original curve and find the minimum squared error approximation by solving:

$$\begin{pmatrix} \hat{v}_1^2 & q_1^2 & \hat{v}_1 & q_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & . & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{v}_n^2 & q_n^2 & \hat{v}_n & q_n^2 & 1 \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_4 \\ c_5 \\ c_6 \end{pmatrix} = \begin{pmatrix} p_1 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ p_n \end{pmatrix}$$

return

Price scenarios: IOHMM

- It is assumed that the market evolves along the time according to discrete market states which are defined by the interaction among:
  - System demand
  - Available generation
  - Strategies of market participants
- For each state, a different relationship is established between these variables and the price:



The pdf conditioned to the set of explanatory variables is different for each state

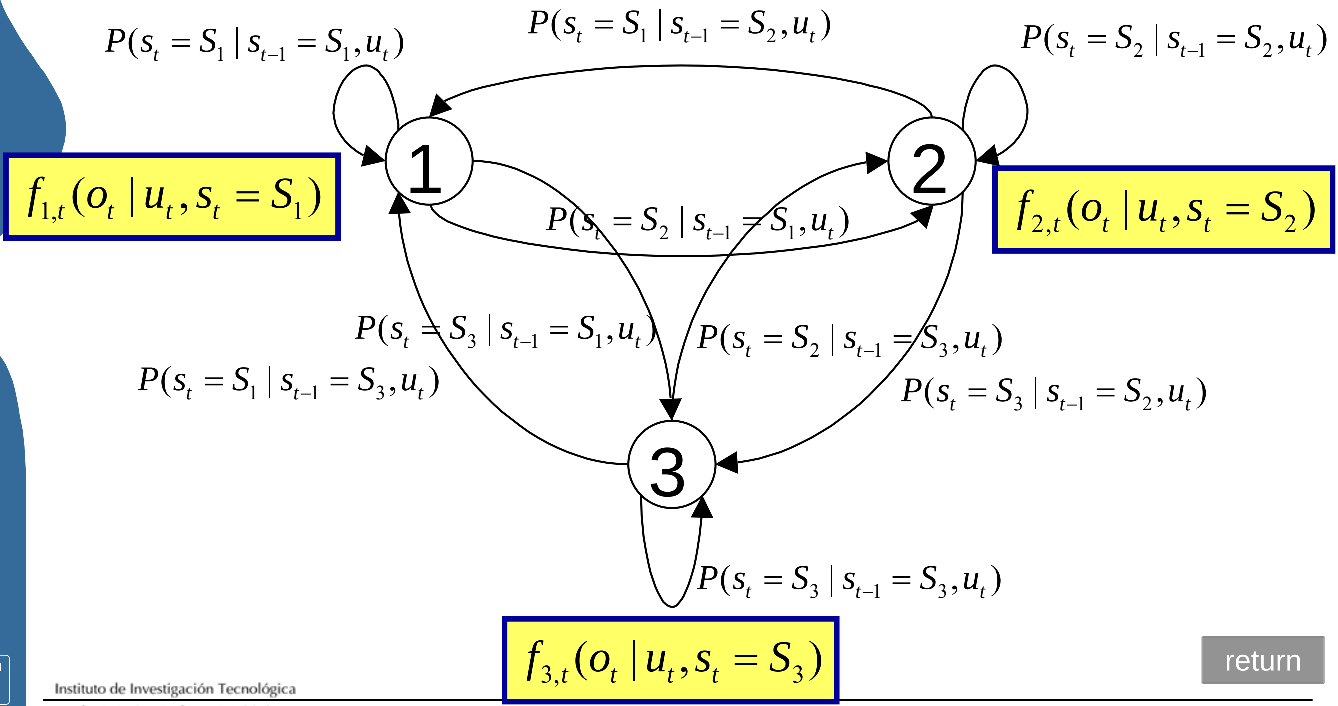
- At each temporal stage, these variables also condition the probability of each state. $P(s_t = S_i \mid u_t)$

States probability conditioned to a set of explanatory variables

return

IOHMM

- Two interconnected process: Markov chain and probability density functions.



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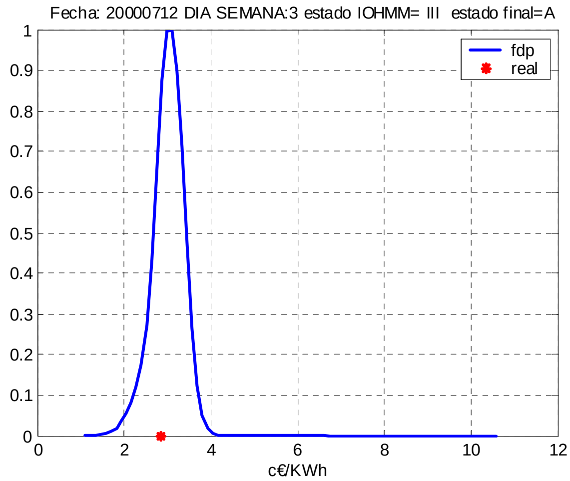
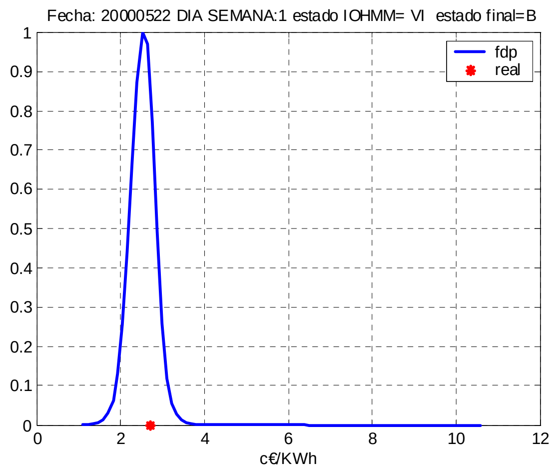
Adjustment of IOHMM. Final states (I)





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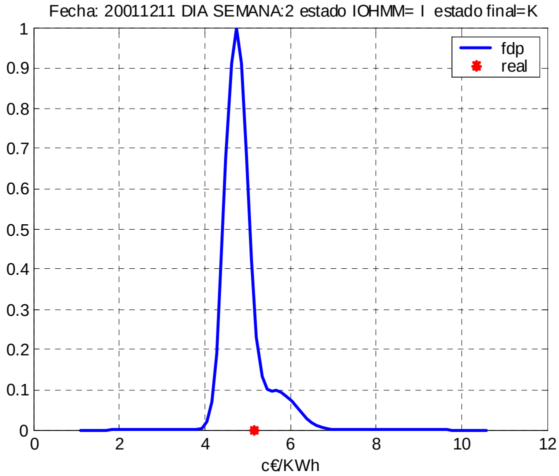
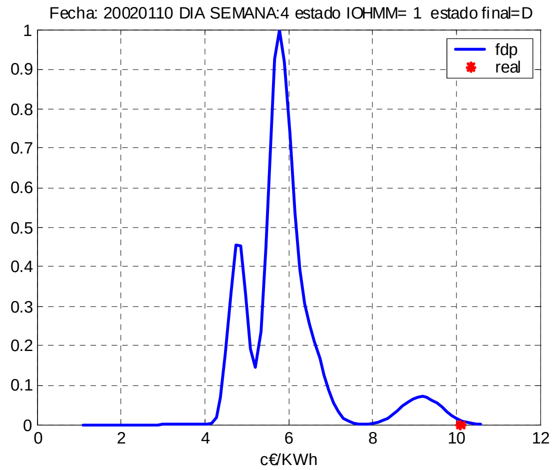
Adjustment of IOHMM. Final states (II)





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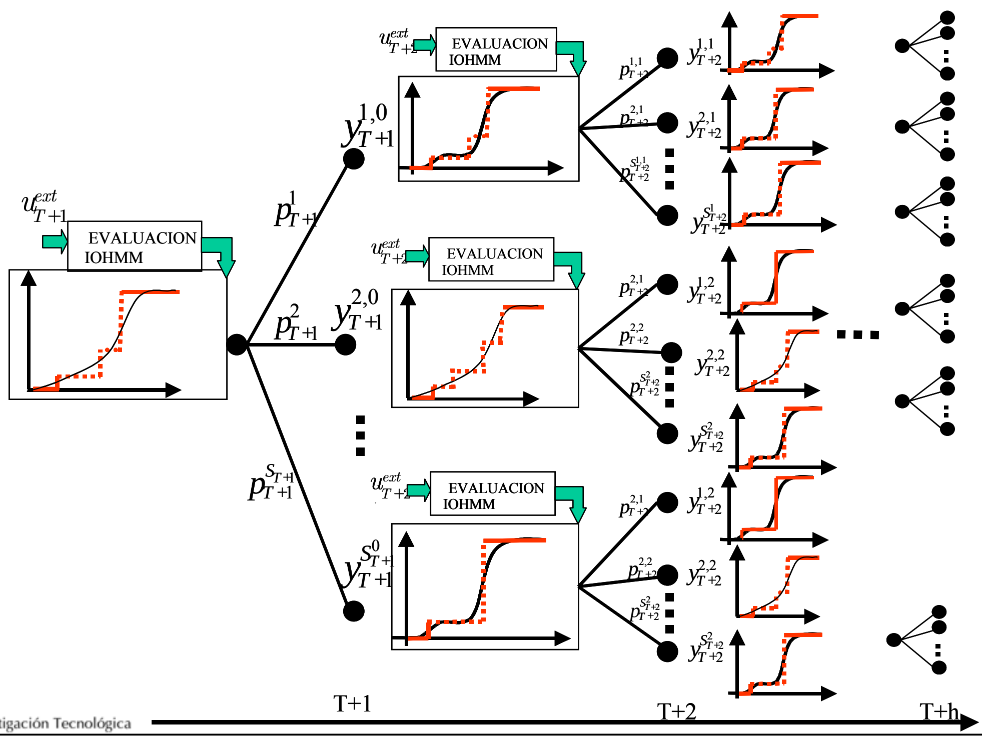
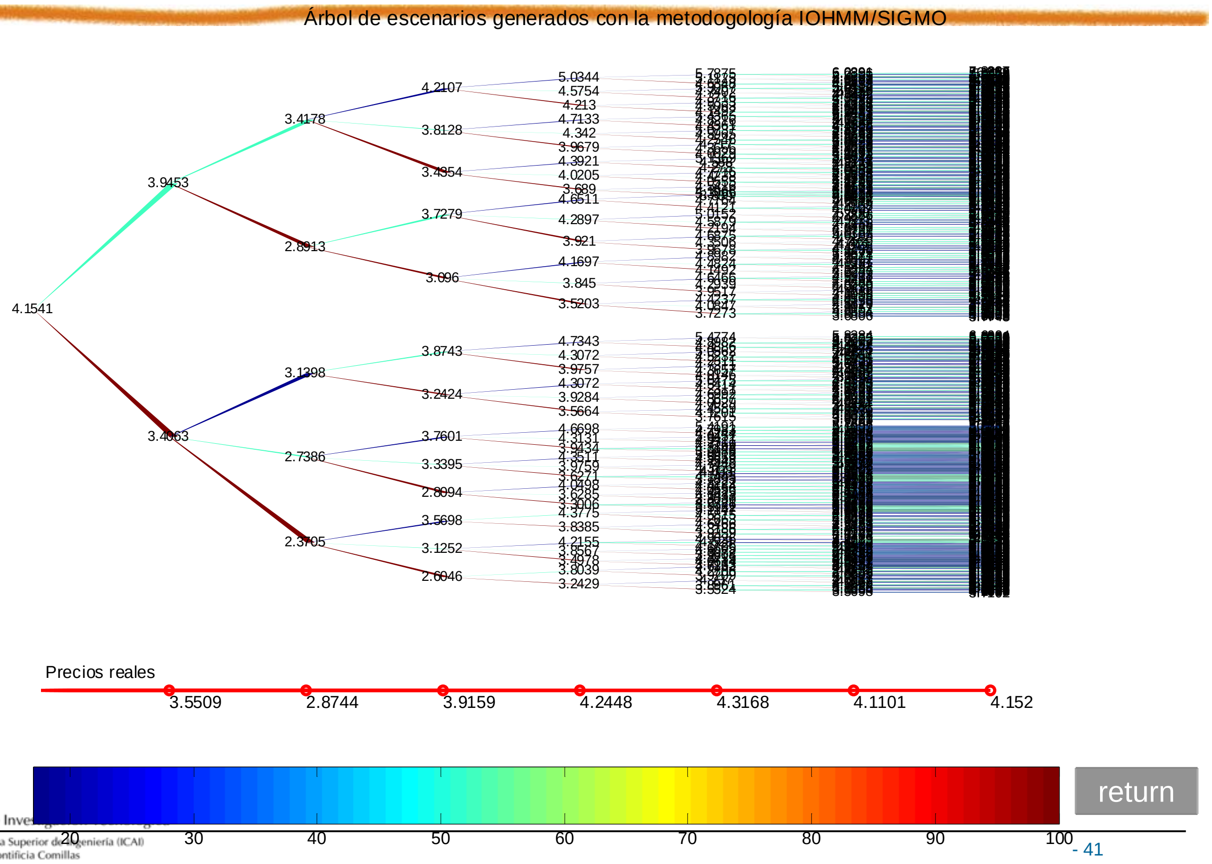
IOHMM scenarios generation (tree)



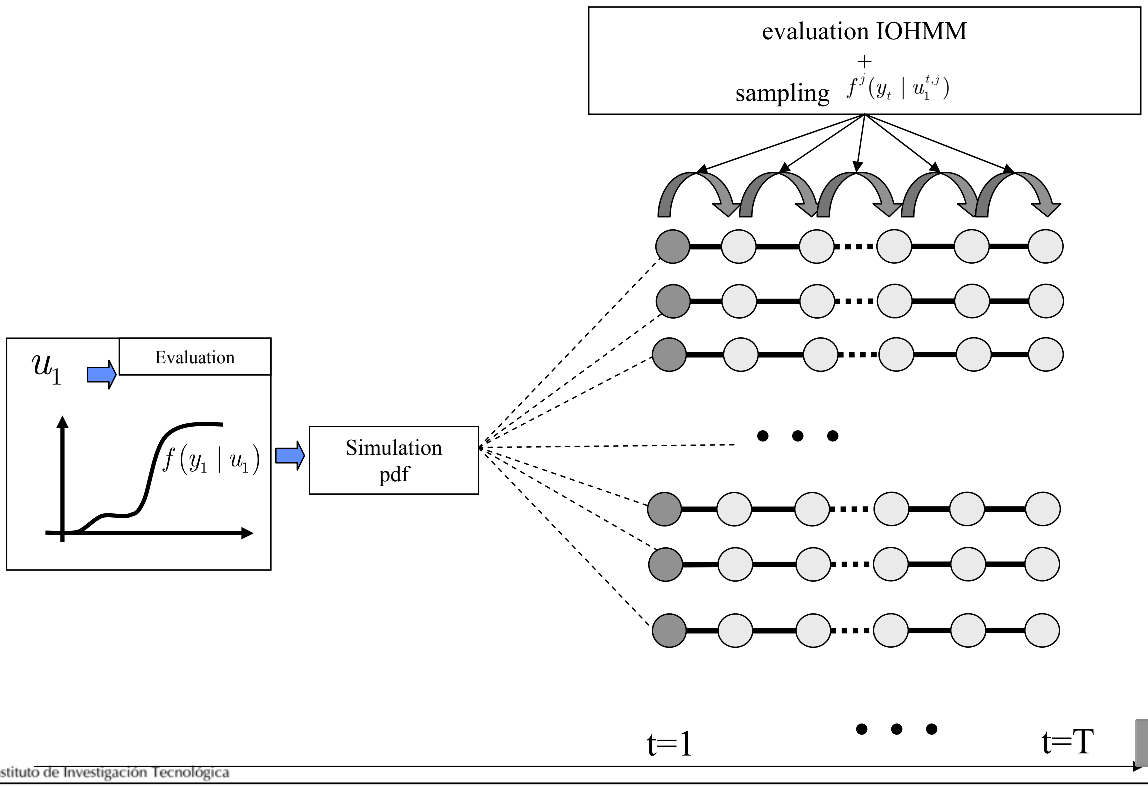
return

Example



return

IOHMM scenarios generation (simulation)



return