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Development of methods for determining demand-limiting setpoint trajectories in buildings using short-term measurements

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Abstract

This paper presents simple approaches for estimating building zone temperature setpoint variations that minimize peak cooling demand during critical demand periods. Three different methods were developed that are termed the semi-analytical (SA), exponential setpoint equation-based semi-analytical (ESA), and load weighted-averaging (WA) methods. The three methods are different in terms of requirements for input data. The SA and ESA methods employ simple inverse building models trained with short-term data and use analytical solutions from the models to determine setpoint trajectories. The WA method is a data-based method in which an optimal weighing factor is found that minimizes a weighted-average of two loads and then used for WA of two initial bound setpoint trajectories. The weighted-averaged setpoint trajectory is adjusted to improve the load shape and can be updated on a daily basis. A companion paper [Lee K-H, Braun JE. Evaluation of methods for determining demand-limiting setpoint trajectories in commercial buildings using short-term measurements. Building and Environment 2007, in press] presents evaluations of the peak load reduction potential associated with implementation of these methods.

Keywords: Building thermal mass; Demand-limiting control; Commercial buildings; Simplified control methods

1. Introduction

There have been a number of simulation and experimental studies that have demonstrated significant potential for reducing peak cooling demand using building thermal mass through control of zone temperatures (e.g., Refs. [1–4]). However, there has been very little work on the development of practical control methods for minimizing peak demand. Lee and Braun [5] developed a model-based demand-limiting method that relies on a detailed inverse model. The method was trained using data from the Energy Resource Station building that houses the Iowa Energy Center and validated experimentally by Lee and Braun [6]. The model-based demand-limiting methodology was tested in the same building and test results showed 30% reductions in peak cooling loads with setpoint adjustments from 70 to 76 °F for a 5-h demand-limiting. These results are consistent with simulation results for this facility. The model-based method described by Lee and Braun [5,6] employs a detailed inverse model that requires a lot of training data and measurements that are not typically available for most buildings (e.g., solar radiation). There is a need for simpler approaches.

Relatively little work has been done in developing simple demand-limiting approaches for adjusting zone temperature setpoints that give near-optimal performance. A simple analytic method that uses a first-order model for the whole building was studied by Rabl and Norford [7]. Ambient temperature and solar radiation were eliminated by taking the difference between modeling equations for two controls, i.e. conventional and setpoint adjustment control. Peak reduction potential was calculated for a building with known building time constants for ‘subcooling’ and ‘warm-up’ periods by assuming energy consumption was constant during the on-peak period. More recently, Braun and Lee [8] developed a simple setpoint equation for demand-limiting from a simple indoor

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Nomenclature

A	area
c	capacitance
c_g	thermal contact factor
c_{pa}	specific heat of air
CC	conventional control
DL	demand-limiting control strategy
d	effective building thickness
$d_{b,eff}$	thickness of shallow mass $= r_c d$
g_m	magnitude factor in approximate equation for radiative heat gain
g_s	shift factor in approximate equation for radiative heat gain
g_t	time lag factor in approximate equation for radiative heat gain
h	convective heat transfer coefficient
k_{dl}	final time stage during the on-peak period
k_t	thermal conductivity
$M_{b,A_{floor}}$	building mass per floor area $= \rho_b d$
N	number
NS	night-setup control strategy
PC	precooling control strategy
$\dot{Q}$	heat transfer rate
$\dot{Q}_b$	rate of instantaneous heat gain to the building air
$\dot{Q}_{b,i,k}$	cooling load for the ith building at time k
$\dot{Q}_{cool,max,i}$	is capacity of the cooling equipment for the ith building
R	thermal resistance
R_a	thermal resistance between zone air and outdoor air
R_d	thermal resistance between shallow mass and deep mass
R_g	thermal resistance between ground and effective entire building mass
R_i	thermal resistance between zone air and effective entire building mass
R_o	thermal resistance between outdoor air and effective entire building mass
R_s	thermal resistance between zone air and shallow mass
r_c	ratio of effective shallow mass capacitance to building capacitance
$r_{A,win,side}$	ratio of window area to building side surface area
T	temperature
t	time
t_{dl}	length of demand-limiting period
V	volume of inside space of building $= A_{floor} ht_{story} N_{story} V_{factor}$
$\dot{V}_{in}$	volume flow rate by infiltration $= \dot{V}_{in,volume} V$
$\dot{V}_{in,volume}$	air exchange rate by infiltration
w	weighting factor

Superscripts

*	optimal
n	nth updated day
p	predicted

Subscripts

a	ambient
adj	setpoint temperature adjustment in WA method
agg	aggregated
avg	average
b	building
cc	conventional control (night setup)
dl	demand-limiting control
eff	effective
env	envelope
f	final state
floor	floor
g	ground
g,r	radiative gain
g,c	convective gain
g,s	solar radiative gain
i	initial state
k	time stage
m	effective building mass
max	maximum
md	deep mass in simple building indoor mass model
ms	shallow mass in simple building indoor mass model
ns	night-setup control
o	outside
oc	occupied period
op	on-peak period
pc	precooling
person	per person
r	roof and ceiling
s	shallow mass
side	side wall of buildings
sp	setpoint
story	story of building
sur	surface
v	ventilation
w	weighted-averaged
win	window
z	building zone air
1	control 1
2	control 2

Greeks

τ	effective time constant in simple exponential setpoint equation
ρ	density

building model. In this approach, effective time constants were determined with a trial-and-error method. Peak load reduction was evaluated through simulation for some representative small commercial buildings. As a fraction of the baseline peak under conventional control, the demand reduction ranged from about 30–100% depending on the climate.

The current paper builds on previous work [5,6] and develops three practical methods for determining demand-limiting setpoint trajectories. The methods differ in terms of implementation requirements and performance. A detailed evaluation of the three approaches is presented in a companion paper [9].

2. Demand-limiting control using building thermal mass

Fig. 1 depicts temperature setpoint changes for demand-limiting control methods that utilize building thermal mass during a critical peak period in the afternoon. In order to precool the structure, building temperature setpoints are set at a lower bound of comfort until the demand-limiting period begins. During the demand-limiting period, the setpoints are adjusted between lower and upper bounds of comfort following a trajectory that minimizes the peak load requirement. Limited test results from Lee and Braun [6] indicate that occupant comfort is not significantly affected when zone temperatures are maintained at 70 °F (21.1 °C) during morning hours and then raised to 78 °F (25.6 °C) during afternoon. Variation of the setpoints controls the rate of heat gains from the interior surfaces and has a profound effect on the load variation with time. Simple methods for setpoint adjustment include 'linear-rise (LR)' and 'step-up (SU)' trajectories that are depicted in Fig. 1. However, these methods have been shown by Lee and Braun [6] not to be optimal for minimizing peak demand. In this paper, three demand-limiting methods for estimating optimal setpoint trajectories are developed. All three methods require short-term load data obtained from buildings during afternoon periods that are characteristic of periods where the demand-limiting will be applied.

3. Demand-limiting methods

3.1. Semi-analytical (SA) method

3.1.1. Model and analysis

The SA method determines an analytical expression for demand-limiting setpoint from a simple building model that characterizes thermal interactions between the interior space and a "shallow" interior mass. The concept of thermal capacitance in the shallow mass was suggested and validated with simple testing for heating and restoration of space with concrete walls [10]. A schematic diagram of the SA method is illustrated in Fig. 2. Actual cooling load data under conventional control are used for estimating parameters associated with a simple building model. The parameters are then used within an analytic expression for the demand-limiting setpoint trajectory.

Fig. 3 depicts the simple interior mass building model that is used for the SA method to describe the thermal behavior of a building over the demand-limiting period. In this figure, T_a is the outdoor air temperature, T_z the zone air temperature, C_{ms} the thermal capacitance of the shallow mass, R_d the thermal resistance between the shallow and deep mass, R_s the thermal resistance between the zone air and shallow mass, R_a the resistance between the indoor air and outdoor air, T_{md} the temperature of the deep mass,

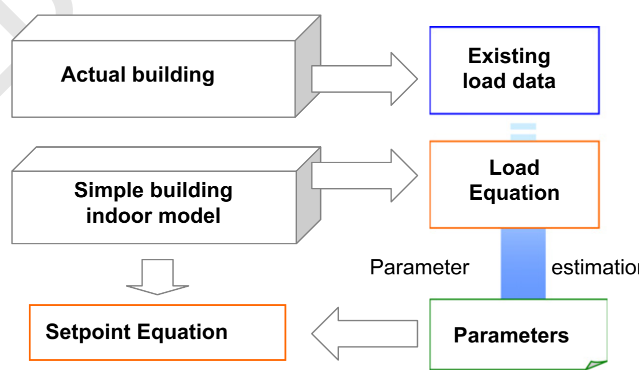

Fig. 2. Schematic illustration of SA method.

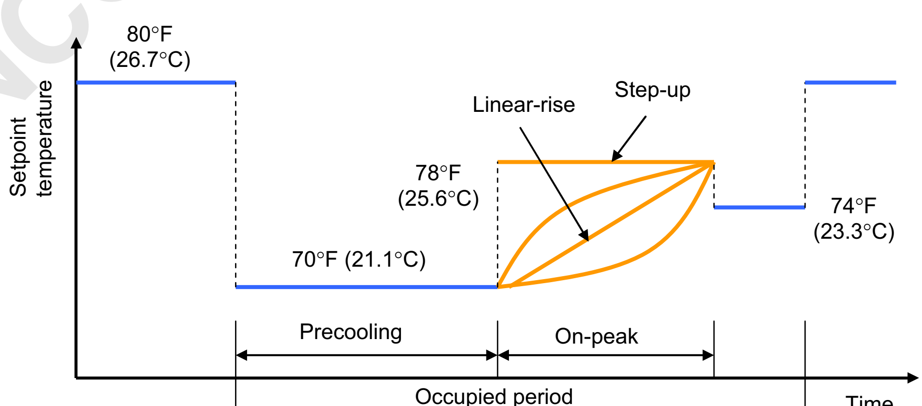

Fig. 1. Example demand-limiting building setpoint temperature controls.

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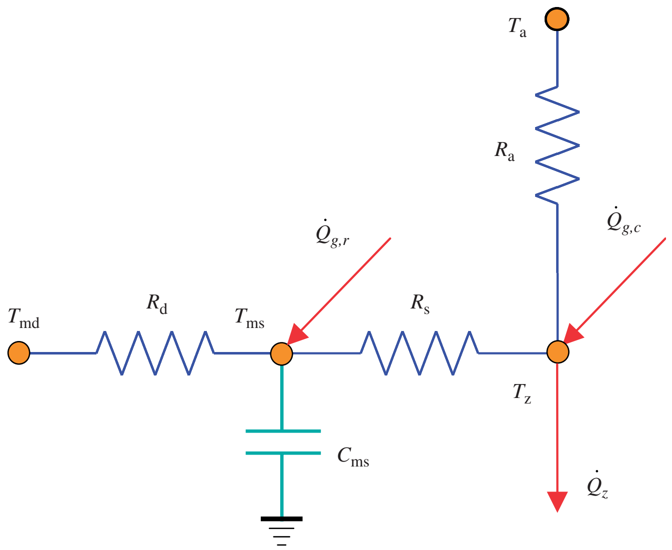

Fig. 3. Simple indoor mass building model for SA method.

T_{ms} the temperature of the shallow mass, $\dot{Q}_{g,c}$ the convective heat gain to the zone air from lighting, equipment, and occupants within the interior spaces, $\dot{Q}_{g,r}$ the radiative heat transfer to the shallow mass surfaces due to internal sources and solar radiation transmitted through windows, and $\dot{Q}_z$ is the zone sensible cooling load. The building network model characterizes the sensible cooling requirement assuming a deep mass temperature is nearly constant over the relatively short demand-limiting period. Radiative heat gain involving transmitted solar radiation into the building space acts on the shallow mass node and convective heat gain occurs to a zone temperature node. Short-term coupling of the zone and outdoor air occurs due to combined effects of conduction heat transfer within the window and convection due to infiltration and ventilation.

From the simple indoor mass building model, two analytic expressions are derived: (1) the cooling load requirement under conventional control and (2) a zone setpoint temperature trajectory for minimizing the peak cooling load during the demand-limiting period. The governing differential equations for the simple indoor building model are:

$$C_{ms}\frac{dT_{ms}}{dt} = \frac{T_{md} - T_{ms}}{R_d} + \frac{T_z - T_{ms}}{R_s} + \dot{Q}_{g,r} \quad (1)$$

$$0 = \frac{T_{ms} - T_z}{R_s} + \frac{T_a - T_z}{R_a} + \dot{Q}_{g,c} - \dot{Q}_z \quad \text{for } 0 \leqslant t \leqslant t_{dl}. \quad (2)$$

In developing the SA method, it was assumed that during the demand-limiting period the outdoor temperature, T_a, can be expressed as a quadratic polynomial function of time and the radiative heat gain, $\dot{Q}_{g,r}$, can be represented using a cubic polynomial variation with time. Radiative heat gain is not directly measured and therefore it was assumed that its time variation is related to the variation in cooling load through a constant multiplication factor, g_m, a constant time lag, g_t, and a constant shift factor, g_s. Equations for the radiative heat gain and outdoor temperature are given in Appendix A.

Parameters associated with Eqs. (1) and (2) are determined using data for the building operating under conventional control with fixed zone setpoint temperatures during the demand-limiting period. Under these conditions, the shallow mass temperature, T_{ms}, in Eqs. (1) and (2) is eliminated and the resulting equation is rearranged to give a first-order differential equation for zone sensible cooling load with fixed zone temperature, $\dot{Q}_{z,cc}$ (see Appendix B). The differential equation is solved using an initial condition of $\dot{Q}_{z,cc}(0) = \dot{Q}_{z,cc,i}$. Appendix B gives the development and resulting analytical expression for the cooling load. The generic dependence of the cooling load on time and building-specific parameters is expressed as

$$\dot{Q}_{z,cc}(t) = f(t : C_s, R_d, R_s, R_a, g_m, g_t, g_s, T_{md,cc}, \dot{Q}_{g,c}) \quad (3)$$

where $T_{md,cc}$ is the deep mass temperature associated with conventional control. The building parameters within the cooling load Eq. (3) are estimated using non-linear regression applied to cooling load data obtained for demand-limiting periods where zone temperature is constant. A constraint with regards to the radiative heat gain is applied to the regression:

$$\dot{Q}_{g,r}(t) \geqslant 0. \quad (4)$$

In order to determine the demand-limiting setpoint trajectory, it is assumed that a constant cooling load is optimal for the demand-limiting period. With $\dot{Q}_z = \dot{Q}_{z,dl} = \text{constant}$, the term T_{ms} from Eqs. (1) and (2) is eliminated and the resulting equation is rearranged for T_z to yield a first-order differential equation for zone temperature. The differential equation is solved to give an analytical expression for zone temperature using an initial condition of $T_{z,dl}(0) = T_{z,i}$ (e.g., a precooling temperature at the lower bound of acceptable comfort). The solution is termed the 'open-ended' demand-limiting setpoint equation to signify that the zone temperature during the demand-limiting period is not constrained. The development and resulting expression are given in Appendix C, whereas the functional dependence is expressed as

$$T_{z,dl}(t) = f(t : C_{ms}, R_d, R_s, R_a, g_m, g_t, g_s, T_{md,dl}, \dot{Q}_{g,c}, \dot{Q}_{z,dl}). \quad (5)$$

A closed-ended form of the demand-limiting equation is obtained by applying a constraint for the setpoint at the end of the demand-limiting period (e.g., the upper limit for acceptable comfort) such that $T_{z,dl}(t_{dl}) = T_{z,f}$. The application of this constraint allows elimination of the deep mass temperature, convective gains and demand-limiting cooling rate. The development and resulting expression are given in Appendix D and the functional dependence is described by

$$T_{z,dl}(t) = f(t : C_{ms}, R_d, R_s, R_a, g_m, g_t, g_s). \quad (6)$$

The closed-ended setpoint equation provides a simple means for estimating a zone temperature setpoint variation

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during the demand-limiting period that results in a constant cooling requirement and is bounded between minimum and maximum limits of comfort.

3.1.2. Approximation of parameters

Parameters in the analytical Eq. (3) for cooling load under conventional control are estimated using non-linear regression with actual data. There are two phases associated with the parameter estimation process as described by Chaturvedi and Braun [11]: global search and local search. The global search uses a systematic search to determine reasonable values of the parameters within bounds determined from a crude building description. The local search uses a local non-linear regression method to further improve the parameter estimates by minimizing the root-mean-squared error between measured and calculated cooling loads for the training duration. The combination of a local and a global phase provides a robust algorithm for determining parameters and only requires minimal preliminary building information.

For the global search phase, building geometry and thermal properties of air and building materials are used to determine lower and upper bounds of thermal parameters: the shallow mass thermal capacitance C_{ms} and thermal resistances R_{d}, R_{s}, and R_{a}. Bounds on the geometry and property parameters are estimated from knowledge of the building. A companion paper [9] provides example bounds for building geometry and property parameters used for a number of different case studies. Equations for converting these parameters to parameters used in Eq. (3) are presented below.

To determine bounds, the shallow mass thermal capacitance can be estimated from:

$$C_{\mathrm{ms}} = r_{\mathrm{c}} M_{\mathrm{b},A_{\mathrm{floor}}} A_{\mathrm{floor}} c_{\mathrm{b}} \tag{7}$$

where A_{floor} is the floor area (m^2), $M_{\mathrm{b},A_{\mathrm{floor}}}$ the building mass per unit of floor area (kg/m^2) $= \rho_{\mathrm{s}} d$, ρ_{s} the density of building material in close contact with the indoor space (kg/m^3), d the effective building thickness (m), c_{b} the specific heat of building envelope (J/kg K), and r_{c} is the ratio of effective shallow mass capacitance to building capacitance.

Thermal resistance between the deep mass and shallow mass is approximated as:

$$R_{\mathrm{d}} = \frac{d_{\mathrm{b,eff}}}{k_{\mathrm{b}} A_{\mathrm{sur,ms}}} \tag{8}$$

where $d_{\mathrm{b,eff}}$ is the thickness of shallow mass $= r_{\mathrm{c}} d$ (m), d the effective building thickness (m), k_{b} the thermal conductivity of building envelope shallow mass (W/m K), $A_{\mathrm{sur,ms}} = A_{\mathrm{sur,env}}$ the surface area of shallow mass, $A_{\mathrm{sur,env}} = A_{\mathrm{side}} + A_{\mathrm{floor}} + A_{\mathrm{roof}}$ the envelope surface area, $A_{\mathrm{side}} = 4[\sqrt{A_{\mathrm{floor}}}\mathrm{ht}_{\mathrm{story}} N_{\mathrm{story}}(1 - r_{\mathrm{A,win,side}})]$ the surface area of four sides of an effective building having a square shape, N_{story} the number of building stories, $\mathrm{ht}_{\mathrm{story}}$ the building height per story (m), $r_{\mathrm{A,win,side}}$ the ratio of window area to building side surface area, $A_{\mathrm{roof}} = A_{\mathrm{floor}}(1 - r_{\mathrm{A,win,roof}})$ the surface area of roof, and $r_{\mathrm{A,win,roof}}$ is the ratio of window to building roof area.

The thermal resistance between the shallow mass and zone air is approximated as:

$$R_{\mathrm{s}} = \frac{1}{h_{\mathrm{i}} A_{\mathrm{sur,ms}}} \tag{9}$$

where h_{i} is the inside convection coefficient (W/m^2 K).

The thermal resistance between the zone air and outdoor air is approximated as:

$$\frac{1}{R_{\mathrm{a}}} = \frac{1}{R_{\mathrm{win}}} + \frac{1}{R_{\mathrm{vent}}} \tag{10}$$

$$R_{\mathrm{win}} = \frac{1}{h_{\mathrm{i}} A_{\mathrm{win}}} + \frac{d_{\mathrm{win}}}{k_{\mathrm{win}} A_{\mathrm{win}}} + \frac{1}{h_{\mathrm{o}} A_{\mathrm{win}}} \tag{11}$$

$$R_{\mathrm{vent}} = \frac{1}{\rho_{\mathrm{a}} c_{\mathrm{pa}} (\dot{V}_{\mathrm{vent}} + \dot{V}_{\mathrm{in}})} \tag{12}$$

where $A_{\mathrm{win}} = 4(\sqrt{A_{\mathrm{floor}}}\mathrm{ht}_{\mathrm{story}} N_{\mathrm{story}} r_{\mathrm{A,win,side}}) + A_{\mathrm{floor}} r_{\mathrm{A,win,roof}}$ is the surface area of windows, d_{win} the window thickness (m), h_{o} the outside convection coefficient (W/m^2 K), k_{win} the window thermal conductivity (W/m K), c_{pa} the specific heat of air (J/kg K), ρ_{a} the density of air (kg/m^3), $\dot{V}_{\mathrm{vent,person}}$ the required ventilation flow rate per person (m^3/h-person), $N_{\mathrm{person,floor}}$ the people number per floor area, $\dot{V}_{\mathrm{in,volume}}$ the air exchange rate by infiltration (1/h), $\dot{V}_{\mathrm{vent}} = \dot{V}_{\mathrm{vent,person}} N_{\mathrm{person,floor}} N_{\mathrm{story}}$ the ventilation flow rate into/out of building (m^3/h), V the volume of inside space of building $= A_{\mathrm{floor}}\mathrm{ht}_{\mathrm{story}} N_{\mathrm{story}} V_{\mathrm{factor}}$ (m^3), and $\dot{V}_{\mathrm{in}} = \dot{V}_{\mathrm{in,volume}} V$ is the volume flow rate by infiltration (m^3/h).

Upper and lower bounds for heat gains and deep mass temperature within Eq. (3) also need to be specified. An upper bound for the internal convective gain is set as the minimum cooling load that occurs for night-setup control during the demand-limiting period. A lower bound is set as some reasonable fraction of the upper bound (e.g., 50%). The deep mass temperature is assumed to be between the zone setpoint temperature for night-setup control and the highest outdoor temperature.

Bounds for the three factors in the radiative heat gain Eq. (A.2) in Appendix A can be set based on a physical understanding. For example, the multiplication factor should have a value that is somewhat smaller or greater than one since the maximum solar radiation may be lower or higher than the highest cooling load during the on-peak period. The time lag between cooling load and internal radiation is typically about 1–2 h. The shifting factor has the same order of magnitude as the cooling load but can be either negative or positive.

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3.2. Exponential setpoint equation-based semi-analytical (ESA) method

3.2.1. Model and analysis

A simple exponential equation for demand-limiting control was derived by Braun and Lee [8] assuming that all driving input conditions are constant during the demand-limiting period.

$$\frac{T_{z,\mathrm{dl}} - T_{z,\mathrm{i}}}{T_{z,\mathrm{f}} - T_{z,\mathrm{i}}} = \frac{1 - \exp(-t/\tau_{\mathrm{eff}})}{1 - \exp(-t_{\mathrm{dl}}/\tau_{\mathrm{eff}})} \tag{13}$$

where $T_{z,\mathrm{dl}}$ is the setpoint temperature, $T_{z,\mathrm{i}}$ the initial temperature at the start of demand-limiting period (e.g., 70 °F (21.1 °C)), $T_{z,\mathrm{f}}$ the temperature at the end of the demand-limiting period (e.g., 78 °F (25.6 °C)), t the time measured from the start of the demand-limiting period, t_{dl} the length of the demand-limiting period, and τ_{eff} is an effective time constant for the setpoint trajectory. This simple exponential was shown by Braun and Lee [8] to be very effective in peak demand reduction with a proper effective time constant. It is important to note that the effective time constant is not a physical characteristic of the building. Rather, it is a parameter that controls the shape of the setpoint trajectory during the demand-limiting period. Simulation results presented by Braun and Lee [8] were obtained for several prototype buildings by estimating effective time constants for peak demand reduction using a trial-and-error method. However, it is desirable to have a general methodology for estimating effective time constants that minimize peak demand using short-term measurements.

The ESA method produces an effective time constant for Eq. (13) that can be used for demand-limiting control, $\tau_{\mathrm{eff,dl}}$ and is illustrated in Fig. 4. The method requires cooling load data for two different control strategies implemented on two different days, $\dot{Q}_{\mathrm{act},1}$ and $\dot{Q}_{\mathrm{act},2}$. The subscripts 1 and 2 indicate two different controls ('control 1' and 'control 2') for the two different days. One of the strategies should be conventional control and the other a simple demand-limiting strategy, such as a 'linear-rise' setpoint strategy. An equation for cooling load difference, $\Delta\dot{Q}_{z,1-2}$, can be obtained analytically from a simple building model. Two sets of load differences, $\Delta\dot{Q}_{\mathrm{act},1-2}$ from actual measured data and $\Delta\dot{Q}_{z,1-2}$ from analytic equations, are compared and used to estimate parameters for a simple building model as depicted in Fig. 4. The parameters are then used to find an effective time constant $\tau_{\mathrm{eff,dl}}$ that minimizes peak demand ($\dot{Q}_{z,\mathrm{dl}}$ in Fig. 4). More details of the method follow in this section.

The parameter estimation is applied to a simple whole building mass model that is depicted in Fig. 5. In this representation, the building mass node is at a temperature of T_{m} and characterizes the entire effective building mass. Solar radiation, $\dot{Q}_{g,s}$ and internal radiative heat gain, $\dot{Q}_{g,r}$, both act on the building mass node. The building mass is also coupled directly to the outdoor air, zone air, and ground. A massless zone air node is connected to the

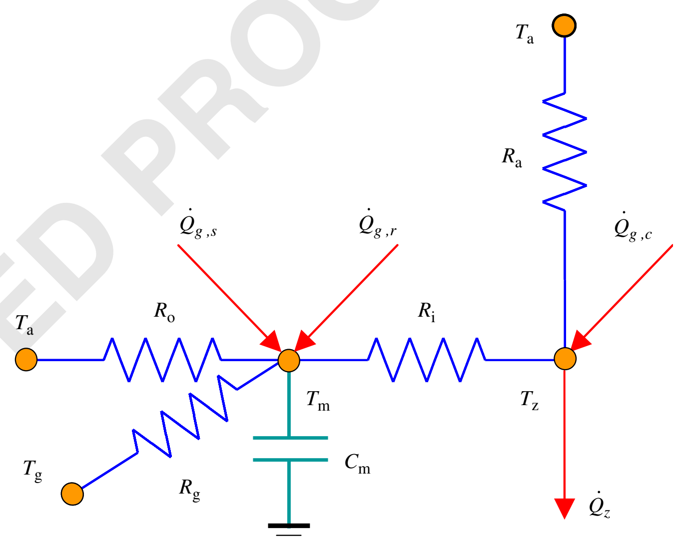

Fig. 5. Simple whole building model for ESA method.

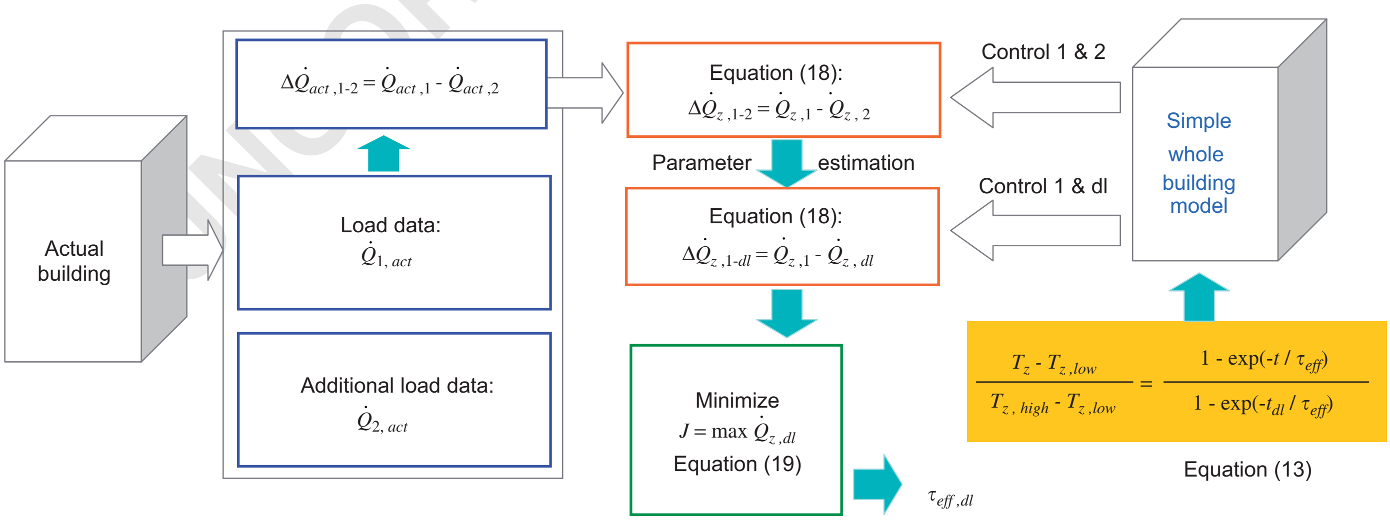

Fig. 4. Overview of ESA method.

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building mass node and outdoor air. The zone air coupling to the outdoor air represents the thermal resistance through windows and convection resulting from infiltration and ventilation. There is also convective heat gain to the zone air from lighting, equipment, and occupants within the interior spaces.

The governing differential equations for this simple whole building model can be written as:

$$C_m \frac{dT_m}{dt} = \frac{T_a - T_m}{R_o} + \frac{T_g - T_m}{R_g} + \frac{T_z - T_m}{R_i} + \dot{Q}_{g,s} + \dot{Q}_{g,r} \tag{14}$$

$$0 = \frac{T_m - T_z}{R_i} + \frac{T_a - T_z}{R_a} + \dot{Q}_{g,c} - \dot{Q}_z \quad 0 < t \leqslant t_{dl} \tag{15}$$

where C_m is the thermal capacitance of the effective building mass, R_o the thermal resistance between the outdoor air and effective building mass, R_i the thermal resistance between the zone air and effective building mass, R_a the thermal resistance between indoor air and outdoor air, T_m the temperature of the effective building mass, T_a the temperature of the outdoor air, $\dot{Q}_{g,c}$ the convective heat gain to the zone air, $\dot{Q}_{g,s}$ the solar radiation on the exterior building walls, $\dot{Q}_{g,r}$ the radiative heat transfer to interior building mass surfaces due to internal sources and solar transmitted through windows, and $\dot{Q}_z$ is the zone sensible cooling load. Eq. (13) arises from the solution to these differential equations with an assumption of constant driving conditions.

In applying the ESA method, it is assumed that all driving conditions, including outdoor temperature, solar radiation, radiative heat gain, and internal convective heat gain, are similar for different afternoon days where demand-limiting would be applied. This assumption eliminates the need to have measurements of actual driving conditions in determining the effective time constant for demand-limiting control. Now consider two different control strategies that employ setpoint trajectories produced with Eq. (13) for time constants $\tau_{eff,1}$ and $\tau_{eff,2}$. Eqs. (14) and (15) apply for each strategy and a set of equations involving differences in state variables are obtained as

$$C_m \frac{d(\Delta T_m)}{dt} = \frac{\Delta T_a - \Delta T_m}{R_o} - \frac{\Delta T_g}{R_g} + \frac{\Delta T_z - \Delta T_m}{R_i} \tag{16}$$

$$0 = \frac{\Delta T_m - \Delta T_z}{R_i} + \frac{\Delta T_a - \Delta T_z}{R_a} - \Delta\dot{Q}_z \tag{17}$$

where $\Delta T_m = T_{m,1} - T_{m,2}$, $\Delta T_z = T_{z,1} - T_{z,2}$, $\Delta T_a = T_{a,1} - T_{a,2}$, and $\Delta\dot{Q}_z = \dot{Q}_{z,1} - \dot{Q}_{z,2}$. The terms involving solar radiation and radiative/convective heat gain have been eliminated. An equation for the outdoor temperature difference is given in Appendix E.

The ΔT_m term in Eqs. (16) and (17) can also be eliminated by combining these equations and then the result can be rearranged to give a first-order differential equation for cooling load difference, $\Delta\dot{Q}_z$. Finally, this differential equation can be solved with an initial condition

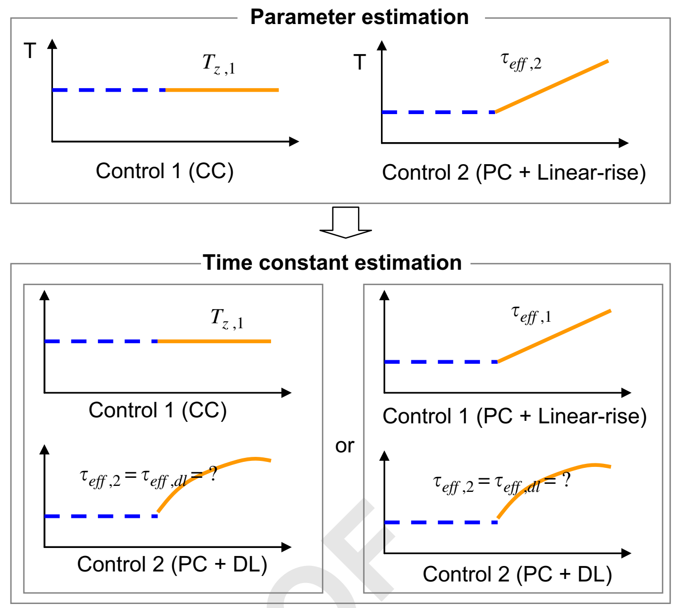

Fig. 6. Two phases in ESA method.

of $\Delta\dot{Q}_z(0) = \dot{Q}_{z,1}(0) - \dot{Q}_{z,2}(0)$. The development and resulting expression are given in Appendix E and the functional dependence can be expressed as

$$\Delta\dot{Q}_z(t) = f(t : \tau_{eff,1}, \tau_{eff,2}, C_m, R_o, R_i, R_a, R_g) \tag{18}$$

As depicted in Fig. 6, the ESA method involves two phases: building model parameter estimation and time constant estimation. The graphs in this figure represent setpoint temperature variations during occupied periods including precooling and demand-limiting periods. In the parameter estimation phase, the parameters of Eq. (18) are estimated using non-linear regression with cooling load difference data for two days having two different control strategies (e.g., conventional control (CC) and precooling with a linear-rise demand-limiting strategy (PC + linear-rise)). Appendix E gives a special-case expression for load difference when one of the strategies is conventional control with T_z set to a constant $T_{z,cc}$. If the second strategy involves a linear-rise in setpoint with control 2, then the time constant $\tau_{eff,2}$ should be set to an artificially large number.

In the time constant estimation phase, one of the two training strategies (either CC or PC + linear-rise) is utilized along with Eq. (18) to determine an effective time constant for a strategy that would minimize the peak cooling load (PC + DL) for a day having similar driving variables. The optimization involves minimizing the following cost function over the demand-limiting period with respect to $\tau_{eff,2}$.

$$J = \max(\dot{Q}_{z,2}(t, \tau_{eff,2})) = \max(\dot{Q}_{z,1}(t) - \Delta\dot{Q}_z(t, \tau_{eff,2})) \quad \text{for } 0 < t \leqslant t_{dl} \tag{19}$$

where $\tau_{eff,2}$ is the effective constant for the demand-limiting strategy (PC + DL in Fig. 6) $\tau_{eff,dl}$, $\dot{Q}_{z,1}$ is measured load for the training strategy ('control 1' in Fig. 6), and $\Delta\dot{Q}_z$ is determined using Eq. (18). The value of J (maximum of

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$\dot{Q}_{z,2}$) that results from this optimization is a prediction of the peak cooling demand under demand-limiting control when the ESA method is applied.

3.2.2. Approximation of thermal parameters

Thermal parameters in the analytical equation for cooling load difference (18) are estimated using non-linear regression with actual load difference data. The parameters are determined using the two-phase search process described for the SA approach that involves a global search and a local search. For the global search phase, bounds for the thermal capacitance of the effective building mass (C_m) and thermal resistances (R_i, R_o, R_g, and R_a) are determined from estimates of bounds for the building geometry and thermal properties of air and building materials as described for the SA method. A companion paper [9] provides example bounds for building geometry and property parameters used for a number of different case studies.

For determining bounds, the effective whole building mass thermal capacitance is approximated as:

$$C_m = M_{b,A_{floor}} A_{floor} c_b \tag{20}$$

where A_{floor} is the floor area (m^2), $M_{b,A_{floor}}$ the building mass per floor area (kg/m^2) $= \rho_b d$, ρ_b the density of the building mass (kg/m^3), d the effective building thickness (m), and c_b is the specific heat of the building envelope (J/kg K).

Thermal resistance between the effective whole building mass and outdoor air is approximated as:

$$R_o = \frac{1}{h_o A_o} \tag{21}$$

where h_o is the outside convection coefficient ($J/h\,m^2\,K$) and A_o is the outside surface area (m^2).

Thermal resistance between the effective whole building mass and zone air is approximated as:

$$R_i = \frac{1}{h_i A_{sur,ms}} \tag{22}$$

where h_i is the inside convection coefficient ($W/m^2\,K$).

Thermal resistance between the zone air and outdoor air is determined using Eqs. (10), (11), and (12). Thermal resistance between the ground and effective building mass is assumed to be:

$$R_g = \frac{c_g}{A_{sur,ms}} \tag{23}$$

where c_g is thermal contact factor.

3.3. Load weighted-averaging (WA) method

3.3.1. Basic WA method

With the WA method, the setpoint trajectory that minimizes the peak cooling load is estimated through a WA of two control setpoint trajectories as depicted in Fig. 7(b). The two setpoint trajectories should produce load variations that intersect at some point during the demand-limiting period as shown in Fig. 7(a). The weighting factor is determined by minimizing the peak of the weight-averaged cooling loads. The optimization problem involves minimizing the following objective function

$$J = \max_{w^*}[w\dot{Q}_{1,k} + (1-w)\dot{Q}_{2,k}] = \max_{w^*}[\dot{Q}_{w,k}] \quad \text{for } 0 < t \leqslant t_{dl} \tag{24}$$

with respect to the weighting factor w, where $\dot{Q}_{1,k}$ is the cooling load for time interval k under control 1, $\dot{Q}_{2,k}$ is the cooling load at time k under control 2, and $\dot{Q}_{w,k}$ is the weighted-averaged cooling load at time k.

The WA method employs the assumption that the cooling load at any time is a linear function of the zone temperature. With this assumption, the zone temperature trajectory that minimizes the peak load is

$$T_{z,w,k} = w^* T_{z,1,k} + (1-w^*) T_{z,2,k} \quad \text{for } 0 < t \leqslant t_{dl} \tag{25}$$

where $T_{z,1,k}$ is the zone setpoint temperature for time interval k with control 1, $T_{z,2,k}$ the zone setpoint temperature for control 2 at time k, and $T_{z,w,k}$ is the optimally weighted-averaged zone setpoint temperature at time k, and w^* is the optimal weighting factor determined by minimizing the cost function in Eq. (24).

The example depicted in Fig. 7 shows a ‘linear-rise’ setpoint variation that results in a decreasing cooling load over the demand-limiting period and a ‘step-up’ setpoint that causes an increasing cooling load. Both setpoint variations have precooling prior to the on-peak time period. The optimal weighting factor determines the WA

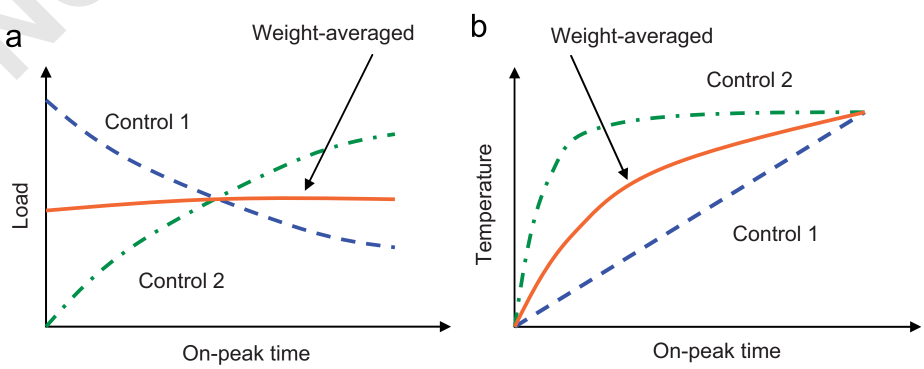

Fig. 7. Schematic illustration of WA method: (a) load and (b) zone temperature.

of these two load profiles that would minimize the peak load. When this weighting factor is applied to zone temperature profiles, a new setpoint trajectory is estimated that is between the two original setpoint trajectories.

Eq. (25) assumes linearity and only employs a single weighting constant to adjust the setpoint trajectory. Therefore, it may produce a cooling load variation that is not flat. In order to improve the shape of the cooling load profile, the setpoints for individual hours within the demand-limiting are adjusted using a local weighting scheme. The adjustment process also employs the assumption of a linear variation of cooling load with zone temperature variation at any time during the demand-limiting period. The adjustment uses the weighted-averaged setpoint trajectory and weighted-averaged load profile to estimate a trajectory that would produce a flat load equal to the average of the weighted-averaged loads. The setpoint trajectory of Eq. (25) that is obtained from the WA is adjusted using the following equation.

$$T_{\mathrm{z,dl},k} = T_{\mathrm{z,w},k} + \Delta T_{\mathrm{adj},k} \tag{26}$$

$$\Delta T_{\mathrm{adj},k} = \frac{\dot{Q}_{\mathrm{w},k} - \dot{Q}_{\mathrm{w,avg}}}{\max|\dot{Q}_{\mathrm{w},k} - \dot{Q}_{\mathrm{w,avg}}|} T_{\mathrm{adj,max}} \tag{27}$$

where $\dot{Q}_{\mathrm{w},k}$ is the weighted-averaged cooling load using $\dot{Q}_{1,k}$ and $\dot{Q}_{2,k}$ at time k, $T_{\mathrm{adj,max}}$ the maximum allowable adjustment temperature for a given hour (e.g., 0.5 or 1.0 °F (0.28 or 0.56 °C)), and $\dot{Q}_{\mathrm{w,avg}}$ is the average of the weighted-averaged cooling load $\dot{Q}_{w,k}$ over the demand-limiting period. The algorithm tends to produce a flat cooling load profile that minimizes differences between the hourly and averaged loads over the demand-limiting period.

In the WA method, two initial days with upper and lower bound load data are assumed to have similar weather conditions. To compensate for different weather conditions, cooling load data are normalized by dividing by the initial cooling load at the start of the demand-limiting period before the method is applied.

3.3.2. Updating WA method

The setpoint trajectory from the basic WA method can be updated on a daily basis so as to improve the shape of the cooling load and respond to changing conditions. The updating process uses the concept of phase cancellation of two functions which are 180° out of phase with each other. Phase cancellation is used primarily in the theory of wave superposition and is sometimes termed destructive interference [12]. If two sets of load data are 180° out of phase, then the optimal weighting factor can be updated perfectly. If a measured load profile for the demand-limiting period is not perfectly flat, then the setpoint trajectory is adjusted to obtain a 180° out-of-phase load profile for phase cancellation. The updating strategy involves using the setpoint trajectory and measured load profile for the most recent demand-limiting period to estimate a trajectory that would produce a 180° out-of-phase load. This trajectory is then implemented and cooling loads are measured. Then, the WA approach is applied to the load data from these two days to determine the new updated demand-limiting trajectory. This process is continually applied for demand-limiting days.

The setpoint trajectory is updated using sequences of two days. First, the basic WA method is applied to determine a setpoint trajectory. On the first day of each updating two-day sequence, the setpoint trajectory is adjusted from the previous days' setpoint trajectory using phase cancellation with a locally linear assumption in a manner very similar to that presented for the basic WA method. The hourly adjustments for phase cancellation are determined on odd days within the updating process as:

$$T^{n}_{\mathrm{z,2},k} = T^{n-1}_{\mathrm{z,1},k} + \Delta T^{n}_{\mathrm{adj},k} \quad (n = 1, 3, 5, \ldots) \tag{28}$$

where

$$\Delta T^{n}_{\mathrm{adj},k} = \frac{\dot{Q}^{n-1}_{1,k} - \dot{Q}^{n-1}_{1,\mathrm{avg}}}{\max|\dot{Q}^{n-1}_{1,k} - \dot{Q}^{n-1}_{1,\mathrm{avg}}|} T^{n}_{\mathrm{adj,max}}, \tag{29}$$

$$T^{n}_{\mathrm{adj,max}} = \frac{\max\{\dot{Q}^{n-1}_{1,k}\} - \dot{Q}^{n-1}_{1,\mathrm{avg}}}{\max\{\dot{Q}^{0}_{1,k}\} - \dot{Q}^{0}_{1,\mathrm{avg}}} T_{\mathrm{adj,max}}, \tag{30}$$

and where n is an index representing the day after the start of the updating process, $T_{\mathrm{adj,max}}$ the maximum allowable adjustment temperature (e.g., 0.5 or 1.0 °F (0.28 or 0.56 °C)), and $T^{n}_{\mathrm{adj,max}}$ is a maximum allowable adjustment temperature for the demand-limiting period on the nth day of updating.

The difference between the hourly adjustment scheme of the updating and basic WA methods is that the maximum allowable adjustment, $T^{n}_{\mathrm{adj,max}}$ varies according to the deviation of the hourly and daily average loads. This tends to dampen the fluctuations in the setpoint trajectory as the load profile approaches the optimum. The determination of the setpoint trajectory for $n = 1$ requires use of the setpoint trajectory determined with the basic WA method, $T^{0}_{\mathrm{z,1},k}$ ($= T_{\mathrm{z,w},k}$ in the basic WA method), and the loads that result from implementation of this trajectory, $\dot{Q}^{0}_{\mathrm{z,1},k}$. The load profile from the basic WA method is also used as a normalization factor in determining a maximum temperature adjustment for each hour within the phase cancellation procedure.

On the second day of each updating two-day sequence, the setpoint trajectory is adjusted from the previous days' setpoint trajectory using WA for the last two demand-limiting days. For each hour within the demand-limiting period on even days within the updating process, the setpoint temperature is determined as a weighted average of the setpoints for the same hour on the previous two demand-limiting days according to

$$T^{n}_{\mathrm{z,1},k} = w^{*}_{n} T^{n-2}_{\mathrm{z,1},k} + (1 - w^{*}_{n}) T^{n-1}_{\mathrm{z,2},k} \quad (n = 2, 4, 6, \ldots) \tag{31}$$

where w^{*}_{n} is the optimal weighting factor determined for the nth day of updating by minimizing the following objective function.

$$J_n = \max_{w_n^*}[w_n \dot{Q}_{1,k}^{n-2} + (1-w_n)\dot{Q}_{2,k}^{n-1}] \quad (n = 2, 4, 6, \ldots) \tag{32}$$

The setpoint trajectory can be continually updated using these two-day sequences of load phase cancellation and WA.

3.4. Application of WA method for building aggregates

Load aggregation for peak demand reduction has some benefits compared to individual building load control such as improvement of load factors[1], possibility of smaller demand charges, and simpler implementation of demand control [14]. Model-based controls require building response models for all of the aggregated buildings to determine optimal setpoint trajectories of each building or a single optimal setpoint trajectory to minimize peak demand of aggregated building loads. Even if a single equivalent model is considered for response modeling of aggregated buildings, it would be quite difficult to obtain feasible parameters for the aggregated building model.

The WA method is a data-based approach that requires no model and can be applied to an aggregated building application if the linearity assumption is valid. The WA method can be adapted to determine a single setpoint trajectory to minimize peak demand of aggregated building loads.

3.4.1. Demand-limiting problem for aggregated building loads

Demand-limiting control for building aggregates is treated as an optimization problem for determining a single setpoint trajectory that minimizes the peak demand of aggregated total cooling demands while maintaining zone temperatures within the comfort temperature range for all of the buildings. The problem involves minimization of the following cost function:

$$J = \max\left\{\sum_{i=1}^{N_b} \dot{Q}_{b,i,k}\right\} \text{for the demand-limiting period} \tag{33}$$

with respect to $T_{z,k}$ subject to $T_{z,i} \leqslant T_{z,k} \leqslant T_{z,f}$ and $0 \leqslant \dot{Q}_{b,i,k} \leqslant \dot{Q}_{cool,max,i}$ where $\dot{Q}_{b,i,k}$ is cooling load for the ith building at time k, $\dot{Q}_{cool,max,i}$ is capacity of the cooling equipment for the ith building and N_b is the number of buildings.

3.4.2. WA method for building aggregates

If the same cooling setpoint is used for all of the buildings in the building aggregate, then aggregated cooling loads can be expressed as a function of the single setpoint.

[1]The load factor was defined as "the ratio of average demand in kW divided by the maximum demand in kW. The average demand is calculated as the ratio of monthly energy use in kWh divided by the number of hours in the month ([13])".

$$\dot{Q}_{agg,k}(T_{z,k}) = \left\{\sum_{i=1}^{N_b} \dot{Q}_{i,k}(T_{z,k})\right\} \tag{34}$$

where $\dot{Q}_{agg,k}(T_{z,k})$ is the total sum of cooling loads at time k for building aggregates as a function of zone setpoint temperature $T_{z,k}$. The zone setpoint temperature can be expressed as a sum of two arbitrary setpoint temperatures $T_{z,a,k}$ and $T_{z,b,k}$ with arbitrary constants a and b.

$$T_{z,k} = aT_{z,a,k} + bT_{z,b,k}. \tag{35}$$

If an individual cooling load at any time is a linear function of the zone temperature, then it can be written as:

$$\dot{Q}_{i,k}(aT_{z,a,k} + bT_{z,b,k}) = a\dot{Q}_{i,k}(T_{z,a,k}) + b\dot{Q}_{i,k}(T_{z,b,k}) \tag{36}$$

If we substitute Eq. (35) into (34), then

$$\begin{aligned}\dot{Q}_{agg,k}(T_{z,k}) &= \left\{\sum_{i=1}^{N_b} \dot{Q}_{i,k}(aT_{z,a,k} + bT_{z,b,k})\right\} \\ &= a\sum_{i=1}^{N_b} \dot{Q}_{i,k}(T_{z,a,k}) + b\sum_{i=1}^{N_b} \dot{Q}_{i,k}(T_{z,b,k})\end{aligned} \tag{37}$$

Eq. (37) can be rewritten as:

$$\dot{Q}_{agg,k}(aT_{z,a,k} + bT_{z,b,k}) = a\dot{Q}_{agg,k}(T_{z,a,k}) + b\dot{Q}_{agg,k}(T_{z,b,k}). \tag{38}$$

From Eq. (38), it is obvious that the sum of cooling loads is also a linear function of zone temperature if individual cooling loads are linear with zone temperature. There is no loss of generality to replace a and b with w and $1-w$, respectively. Based on this linearity assumption for the aggregated cooling load, the WA in the WA method can be applied to building aggregates to find a single setpoint trajectory that minimizes the peak aggregated cooling load. The weighting factor is determined by minimizing the peak of the weight-averaged cooling loads for building aggregates. The optimization problem involves minimizing the following objective function

$$J = \max_{w^*}\left\{w\sum_{i=1}^{N_b} \dot{Q}_{1,i,k} + (1-w)\sum_{i=1}^{N_b} \dot{Q}_{2,i,k}\right\} \text{for all } k \text{ in the demand-limiting period} \tag{39}$$

with respect to the weighting factor w, where $\dot{Q}_{1,i,k}$ is the cooling load of the ith building for time interval k under control 1 and $\dot{Q}_{2,i,k}$ is the cooling load of ith building at time k under control 2. With the linearity assumption, the zone temperature trajectory that minimizes the aggregated peak load is

$$T_{z,w,k} = w^* T_{z,1,k} + (1-w^*)T_{z,2,k} \quad \text{for the demand-limiting period} \tag{40}$$

where $T_{z,1,k}$ is the zone setpoint temperature for time interval k with control 1, $T_{z,2,k}$ the setpoint temperature for control 2 at time k, $T_{z,w,k}$ is the optimal zone setpoint temperature at time k, and w^* is the optimal weighting factor determined by minimizing the cost function in Eq. (39). The weighted-averaged setpoint trajectory $T_{z,w,k}$ is

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adjusted using Eqs. (26) and (27). The same method for the updating of the setpoint trajectory used for the individual building approach is used for application to building aggregates.

4. Summary

In this paper, practical methods that use short-term measurement data for determining demand-limiting control setpoint trajectories are described. Three demand-limiting methods, termed SA, ESA (ESA-based SA), and WA, have been developed that have different data requirements. Each method yields an estimate of a building-specific setpoint trajectory that gives a “flat” cooling load profile during a specified demand-limiting period and requires short-term measurements for training.

Both the SA and ESA methods use analytical equations obtained from simple building models and use test data for parameter estimation, while the WA method uses WA of load data. The SA method requires the least data and a strategy can be determined with one day of load data for conventional control. The ESA method requires one additional day of test data compared to the SA method. The WA method requires two test days with setpoint trajectories that bound the optimal solution. Application of the WA method for building aggregates was also presented that uses a single demand-limiting setpoint trajectory to minimize peak demand of aggregated building loads.

A companion paper by Lee and Braun [9] evaluates the performance of these three methods in terms peak load reduction potential for a number of different case studies. The methods require less field data and few inputs than previous methods [5,6] and are very effective in terms of peak demand reduction.

Appendix A. Approximation of radiative heat gain and outdoor temperature

For characterizing the radiative gain profile, cooling load data under conventional control is expressed in the form of cubic polynomials as:

$$\dot{Q}_{\mathrm{z}} = q_0 + q_1 t + q_2 t^2 + q_3 t^3. \tag{A.1}$$

Coefficients in the polynomial equation are obtained using regression of actual load data. It is assumed that the radiative heat gain has a similar shape as the cooling load and can be expressed as a cubic polynomial. The radiative heat gain is assumed to be related to the cooling load data through three parameters: a multiplication factor g_{m}, a constant time lag g_{t}, and a constant shift factor g_{s}.

$$\begin{aligned}\dot{Q}_{\mathrm{g,r}} &= g_0 + g_1 t + g_2 t^2 + g_3 t^3 \\ &= g_{\mathrm{m}}[q_0 + q_1(t+g_{\mathrm{t}}) + q_2(t+g_{\mathrm{t}})^2 + q_3(t+g_{\mathrm{t}})^3] + g_{\mathrm{s}}\end{aligned} \tag{A.2}$$

Then, the coefficients of g_0, g_1, g_2, and g_3 are written as

$$g_0 = g_{\mathrm{s}} + g_{\mathrm{m}}(q_0 + g_{\mathrm{t}} q_1 + g_{\mathrm{t}}^2 q_2 + g_{\mathrm{t}}^3 q_3) \tag{A.3}$$

$$g_1 = g_{\mathrm{m}}(q_1 + 2g_{\mathrm{t}} q_2 + 3g_{\mathrm{t}}^2 q_3) \tag{A.4}$$

$$g_2 = g_{\mathrm{m}}(q_2 + 3g_{\mathrm{t}} q_3) \text{ and } g_3 = g_{\mathrm{m}}(q_3). \tag{A.5}$$

The outdoor temperature variation for the demand-limiting period is expressed as a quadratic polynomial equation. Coefficients in the polynomial equation are obtained using regression of actual outdoor temperature data.

$$T_{\mathrm{a}}(t) = T_{\mathrm{a0}} + T_{\mathrm{a1}} t + T_{\mathrm{a2}} t^2 \tag{A.6}$$

Appendix B. Cooling load equation

The differential equation for cooling load under conventional control is

$$\frac{\mathrm{d}\dot{Q}_{\mathrm{z}}(t)}{\mathrm{d}t} = -\frac{1}{A_1}\dot{Q}_{\mathrm{z}}(t) + \frac{1}{A_2}\frac{\mathrm{d}T_{\mathrm{a}}(t)}{\mathrm{d}t} + \frac{1}{A_3}T_{\mathrm{a}}(t) + \frac{1}{A_4}\dot{Q}_{\mathrm{g,r}}(t) + K_{\mathrm{cc}} \tag{B.1}$$

where

$$\begin{aligned}K_{\mathrm{cc}} &= \frac{1}{C_{\mathrm{ms}}R_{\mathrm{s}}}\left[\frac{T_{\mathrm{md}}}{R_{\mathrm{d}}} + \frac{T_{\mathrm{z,cc}}}{R_{\mathrm{s}}} - \left(1+\frac{R_{\mathrm{s}}}{R_{\mathrm{a}}}\right)\left(\frac{1}{R_{\mathrm{d}}}+\frac{1}{R_{\mathrm{s}}}\right)T_{\mathrm{z,cc}} \right. \\ &\quad \left. + R_{\mathrm{s}}\left(\frac{1}{R_{\mathrm{d}}}+\frac{1}{R_{\mathrm{s}}}\right)\dot{Q}_{\mathrm{g,c}}\right], \ \frac{1}{A_1} = \frac{1}{R_{\mathrm{d}}C_{\mathrm{ms}}} + \frac{1}{R_{\mathrm{s}}C_{\mathrm{ms}}}, \ \frac{1}{A_2} \\ &= \frac{1}{R_{\mathrm{a}}}, \ \frac{1}{A_3} = \frac{1}{R_{\mathrm{a}}}\left(\frac{1}{R_{\mathrm{d}}C_{\mathrm{ms}}} + \frac{1}{R_{\mathrm{s}}C_{\mathrm{ms}}}\right),\end{aligned}$$

and

$$\frac{1}{A_4} = \frac{1}{R_{\mathrm{s}}C_{\mathrm{ms}}}.$$

The solution of the differential equation for cooling load under conventional control with an initial condition me $\dot{Q}_{\mathrm{z,cc}}(0) = \dot{Q}_{\mathrm{z,cc},i}$ is

$$\begin{aligned}\dot{Q}_{\mathrm{z,cc}}(t) &= \dot{Q}_{\mathrm{z,cc},i}\exp\left(-\frac{t}{A_1}\right) + \frac{1}{2}F_1\left[1-\exp\left(-\frac{t}{A_1}\right)\right] \\ &\quad + F_2 t^3 + F_3 t^2 + F_4 t\end{aligned} \tag{B.2}$$

where

$$\begin{aligned}F_1 = \frac{1}{A_2A_3A_4}[&(2A_1A_2A_4)T_{\mathrm{a0}} + 2(A_1A_3A_4 - A_2A_4A_1^2)T_{\mathrm{a1}} \\ &+ 4(A_2A_4A_1^3 - A_3A_4A_1^2)T_{\mathrm{a2}} + 2(A_1A_2A_3)g_0 \\ &- 2(A_2A_3A_1^2)g_1 + 4(A_2A_3A_1^3)g_2 - 12(A_2A_3A_1^4)g_3 \\ &+ 2(A_1A_2A_3A_4)K_{\mathrm{dl}}],\end{aligned}$$

$$F_2 = \frac{A_1}{A_4}g_3,$$

$$F_3 = \frac{1}{A_2A_3A_4}[(A_1A_2A_3)g_2 - 3(A_1^2A_2A_3)g_3 + (A_1A_2A_4)T_{\mathrm{a2}}],$$

and

$$F_4 = \frac{1}{A_2A_3A_4}[(A_1A_2A_4)T_{a1} + 2(A_1A_3A_4 - A_1^2A_2A_4)T_{a2} + (A_1A_2A_3)g_1 - 2(A_1^2A_2A_3)g_2 + 6(A_1^3A_2A_3)g_3]$$

Appendix C. Open-ended demand-limiting setpoint equation

The differential equation for the setpoint temperature under demand-limiting control is

$$\frac{dT_z(t)}{dt} = -\frac{1}{B_1}T_z(t) + \frac{1}{B_2}\frac{dT_a(t)}{dt} + \frac{1}{B_3}T_a(t) + \frac{1}{B_4}\dot{Q}_{g,r}(t) + K_{dl} \tag{C.1}$$

where

$$K_{dl} = \frac{R_a}{C_{ms}(R_a + R_s)}\left[\frac{T_{md,dl}}{R_d} - R_s\left(\frac{1}{R_d} + \frac{1}{R_s}\right)(\dot{Q}_{z,dl} - \dot{Q}_{g,c})\right],$$

$$\frac{1}{B_1} = \frac{1}{R_dC_{ms}} + \frac{1}{(R_a + R_s)C_{ms}},$$

$$\frac{1}{B_2} = \frac{R_s}{R_a + R_s},$$

$$\frac{1}{B_3} = \frac{R_s}{R_a + R_s}\left(\frac{1}{R_dC_{ms}} + \frac{1}{R_sC_{ms}}\right),$$

and

$$\frac{1}{B_4} = \frac{1}{C_{ms}}\left(\frac{R_a}{R_a + R_s}\right).$$

Solution of the differential equation for the demand-limiting setpoint temperature with an initial condition $T_{z,dl}(0) = T_{z,i}$ is

$$T_{z,dl}(t) = T_{z,i}\exp\left(-\frac{t}{B_1}\right) + \frac{1}{2}F_1\left[1 - \exp\left(-\frac{t}{B_1}\right)\right] + F_2t^3 + F_3t^2 + F_4t \tag{C.2}$$

where

$$F_1 = \frac{1}{B_2B_3B_4}[(2B_1B_2B_4)T_{a0} + 2(B_1B_3B_4 - B_2B_4B_1^2)T_{a1} + 4(B_2B_4B_1^3 - B_3B_4B_1^2)T_{a2} + 2(B_1B_2B_3)g_0 - 2(B_2B_3B_1^2)g_1 + 4(B_2B_3B_1^3)g_2 - 12(B_2B_3B_1^4)g_3 + 2(B_1B_2B_3B_4)K_{dl}],$$

$$F_2 = \frac{B_1}{B_4}g_3,$$

$$F_3 = \frac{1}{B_2B_3B_4}[(B_1B_2B_3)g_2 - 3(B_1^2B_2B_3)g_3 + (B_1B_2B_4)T_{a2}],$$

and

$$F_4 = \frac{1}{B_2B_3B_4}[(B_1B_2B_4)T_{a1} + 2(B_1B_3B_4 - B_1^2B_2B_4)T_{a2} + (B_1B_2B_3)g_1 - 2(B_1^2B_2B_3)g_2 + 6(B_1^3B_2B_3)g_3].$$

Appendix D. Closed-ended demand-limiting setpoint equation

A closed-ended form of the demand-limiting equation is obtained by applying a constraint for the setpoint at the end of the demand-limiting period (e.g., the upper limit for acceptable comfort) such that $T_{z,dl}(t_{dl}) = T_{z,f}$. The application of this constraint allows elimination of the deep mass temperature $T_{md,dl}$, convective gains $\dot{Q}_{g,c}$ and demand-limiting cooling rate $\dot{Q}_{z,dl}$. If the final condition $T_{z,dl}(t_{dl}) = T_{z,f}$ is applied to the open-ended demand-limiting setpoint equation and the equation is re-arranged, then the following setpoint Eq. (D.1) can be obtained. It is termed the closed-ended form of the demand-limiting setpoint equation. It should be noted that the variable F_1, which includes the terms T_{a0}, g_0, $T_{md,dl}$, $\dot{Q}_{z,dl}$, and $\dot{Q}_{g,c}$, does not appear in this equation.

$$\frac{T_{z,dl}(t) - T_{z,i}}{T_{z,f} - T_{z,i}} = \frac{1 - \exp(-t/B_1)}{1 - \exp(-t_{dl}/B_1)} + \frac{F_4}{T_{z,f} - T_{z,i}} \times \left[t - t_{dl}\frac{1 - \exp(-t/B_1)}{1 - \exp(-t_{dl}/B_1)}\right] + \frac{F_3}{T_{z,f} - T_{z,i}} \times \left[t^2 - t_{dl}^2\frac{1 - \exp(-t/B_1)}{1 - \exp(-t_{dl}/B_1)}\right] + \frac{F_2}{T_{z,f} - T_{z,i}} \times \left[t^3 - t_{dl}^3\frac{1 - \exp(-t/B_1)}{1 - \exp(-t_{dl}/B_1)}\right] \tag{D.1}$$

Appendix E. Equations of outdoor temperature difference and load difference

Outdoor temperature difference terms for the two days corresponding to the control 1 and control 2 can be expressed as quadratic polynomial equations.

$$\Delta T_a = (T_{a1,0} + T_{a1,1}t + T_{a1,2}t^2) - (T_{a2,0} + T_{a2,1}t + T_{a2,2}t^2) = (T_{a1,0} - T_{a2,0}) + (T_{a1,1} - T_{a2,1})t + (T_{a1,2} - T_{a2,2})t^2 = \Delta T_{a0} + \Delta T_{a1}t + \Delta T_{a2}t^2 \tag{E.1}$$

where $T_{a1,0}$, $T_{a1,1}$, and $T_{a1,2}$ are coefficients of the outdoor temperature equation for the 'control 1' day, and $T_{a2,0}$, $T_{a2,1}$, and $T_{a2,2}$ are coefficients of the outdoor temperature equation for the 'control 2' day.

The governing differential equation between load difference resulting from application of two different control strategies, 'control 1' and 'control 2', is

$$\frac{d(\Delta\dot{Q}_z)}{dt} = -A_1(\Delta\dot{Q}_z) + A_2(\Delta T_z) - A_3\frac{d(\Delta T_z)}{dt} + A_4(\Delta T_a) + A_5\frac{d(\Delta T_a)}{dt} \tag{E.2}$$

where

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$$A_1 = \frac{1}{C_m}\left(\frac{1}{R_o} + \frac{1}{R_i} + \frac{1}{R_g}\right),$$

$$A_2 = \frac{1}{C_m R_i}\left[\frac{1}{R_i} - \left(\frac{1}{R_o} + \frac{1}{R_i} + \frac{1}{R_g}\right)\left(\frac{R_a + R_i}{R_a}\right)\right],$$

$$A_3 = \frac{R_a + R_i}{R_a R_i},$$

$$A_4 = \frac{1}{C_m R_i}\left[\frac{1}{R_o} + \left(\frac{1}{R_o} + \frac{1}{R_i} + \frac{1}{R_g}\right)\left(\frac{R_i}{R_a}\right)\right],$$

and

$$A_5 = \frac{1}{R_a}.$$

For the zone temperature difference ΔT_z term, two special cases are considered according to the zone temperature setpoint for the control 1 day. The first case is when control 1 is the conventional control with a constant zone temperature.

$$\Delta T_z = T_{z,cc} - \left(\frac{1 - e^{-t/\tau_2}}{1 - e^{-t_{dl}/\tau_2}}\right)(T_{z,2,f} - T_{z,2,i}) + T_{z,2,i} \tag{E.3}$$

where $T_{z,cc}$ is constant zone temperature in conventional control for 'control 1', τ_2 the time constant in the simple exponential equation for 'control 2', $T_{z,2,f}$ the higher bound temperature for 'control 2' during the demand-limiting period, and $T_{z,2,i}$ is the lower bound temperature for 'control 2' during the demand-limiting period. The second case is for when the building is precooled and the demand-limiting setpoint temperature follows the simple exponential Eq. (13) for both 'control 1' and 'control 2'.

$$\Delta T_z = \left[\left(\frac{1 - e^{-t/\tau_1}}{1 - e^{-t_{dl}/\tau_1}}\right)(T_{z,1,f} - T_{z,1,i}) + T_{z,1,i}\right] - \left[\left(\frac{1 - e^{-t/\tau_2}}{1 - e^{-t_{dl}/\tau_2}}\right)(T_{z,2,f} - T_{z,2,i}) + T_{z,2,i}\right] \tag{E.4}$$

The solution of Eq. (E.2) can be expressed in different forms according to the condition of the zone temperature difference term ΔT_z. Firstly, the solution with an initial condition $\Delta\dot{Q}_z(0) = \Delta\dot{Q}_{z,i}$ and zone temperature difference term of (E.3) is

$$\Delta\dot{Q}_z(t) = \Delta\dot{Q}_{z,i}\exp(-A_1 t) + B_1 + B_2 + \frac{B_3 + B_4 + B_5(A_1\tau_2 - 1)}{B_6} \tag{E.5}$$

where

$$B_1 = \frac{e^{-A_1 t}(A_4\Delta T_{a1} + 2A_5\Delta T_{a2}) - 2\Delta T_{a2}(A_4 t + A_5) - A_4\Delta T_{a1}}{A_1^2},$$

$$B_2 = \frac{(A_4\Delta T_{a0}A_1^2 + 2A_4\Delta T_{a2} + A_2 T_{z,cc}A_1^2)(1 - e^{-A_1 t})}{A_1^3},$$

$$B_3 = e^{-t/\tau_2}A_1(T_{z,2,f} - T_{z,2,i})(A_2\tau_2 + A_3),$$

$$B_4 = e^{-A_1 t}(\tau_2 T_{z,2,i}A_1A_2 + A_5\Delta T_{a1} - \Delta T_{a1}A_5A_1\tau_2 - T_{z,2,f}A_1A_3 + T_{z,2,i}A_1A_3 - A_2T_{z,2,f}),$$

$$B_5 = e^{-t_{dl}/\tau_2}[A_2T_{z,2,i} - \Delta T_{a2}t(A_4t + 2A_5) + e^{-A_1 t}(\Delta T_{a1}A_5 - T_{z,2,i}A_2) - \Delta T_{a1}(A_4t + A_5)] + [\Delta T_{a2}t(A_4t + 2A_5) + \Delta T_{a1}(A_4t + A_5) - A_2T_{z,2,f}],$$

and

$$B_6 = A_1(A_1\tau_2 - 1)(1 - e^{-t_{dl}/\tau_2}).$$

Secondly, the solution of Eq. (E.2) with an initial condition $\Delta\dot{Q}_z(0) = \Delta\dot{Q}_{z,i}$ and zone temperature difference term expressed as (E.4) with $T_{z,1,i} = T_{z,2,i} = T_{z,i}$ and $T_{z,1,f} = T_{z,2,f} = T_{z,f}$ is

$$\Delta\dot{Q}_z(t) = B_1 + B_2 + \frac{[e^{-A_1 t}(B_3) + B_4 + B_5(\tau_1A_1 - 1) + B_6]}{B_7} \tag{E.6}$$

where

$$B_1 = \frac{-A_4\Delta T_{a1} + e^{-A_1 t}(A_4\Delta T_{a1} + 2A_5\Delta T_{a2}) - 2\Delta T_{a2}(A_4t + A_5)}{A_1^2},$$

$$B_2 = \frac{A_4(2\Delta T_{a2} + \Delta T_{a0}A_1^2)(1 - e^{-A_1 t})}{A_1^3},$$

$$\begin{aligned}B_3 = {} & (\Delta\dot{Q}_{z,i}A_1^2\tau_1 - A_5\Delta T_{a1}\tau_1A_1 + T_{z,f}A_3A_1 - \Delta\dot{Q}_{z,i}A_1 \\ & - T_{z,i}A_3A_1 - A_2T_{z,i} + A_5\Delta T_{a1} + A_2T_{z,f})(\tau_2A_1 - 1)e^{-t_{dl}/\tau_2} \\ & + [(\tau_2A_1 - 1)(A_5\Delta T_{a1} - \Delta\dot{Q}_{z,i}A_1)e^{(-t_{dl}/\tau_1)-(-t_{dl}/\tau_2)} \\ & + (-A_5\Delta T_{a1}\tau_2A_1 + T_{z,i}A_3A_1 - T_{z,f}A_3A_1 + \Delta\dot{Q}_{z,i}A_1^2\tau_2 \\ & + A_2T_{z,i} - A_2T_{z,f} - \Delta\dot{Q}_{z,i}A_1 + A_5\Delta T_{a1})e^{-t_{dl}/\tau_1} + (A_5\Delta T_{a1} - \\ & \Delta\dot{Q}_{z,i}A_1)(\tau_2A_1 - 1)](\tau_1A_1 - 1) + A_1(T_{z,f} - T_{z,i})(\tau_1 - \tau_2) \\ & (A_3A_1 + A_2),\end{aligned}$$

$$B_4 = e^{-t/\tau_2}A_1(1 - e^{-t_{dl}/\tau_2})(\tau_2A_1 - 1)(T_{z,f} - T_{z,i})(\tau_1A_2 + A_3),$$

$$\begin{aligned}B_5 = {} & e^{-t_{dl}/\tau_1}\{e^{-t/\tau_2}A_1(\tau_2A_2 + A_3)(T_{z,f} - T_{z,i}) \\ & + (\tau_2A_1 - 1)[-A_2(T_{z,f} - T_{z,i}) + \Delta T_{a1}(A_4t + A_5) \\ & + \Delta T_{a2}t(2A_5 + A_4t)]\} + e^{-t/\tau_2}A_1(\tau_2A_2 + A_3) \\ & \times (T_{z,i} - T_{z,f}) + (\tau_2A_1 - 1)[e^{-t_{dl}/\tau_2}(A_2T_{z,f} + \Delta T_{a1}A_4t \\ & + 2\Delta T_{a2}A_5t - A_2T_{z,i} + \Delta T_{a2}A_4t^2 + A_5\Delta T_{a1}) \\ & - e^{(-t_{dl}/\tau_1)-(t_{dl}/\tau_2)}(\Delta T_{a1}A_4t + A_5\Delta T_{a1} + 2\Delta T_{a2}A_5t \\ & + \Delta T_{a2}A_4t^2) - \Delta T_{a2}t(2A_5 + A_4t)],\end{aligned}$$

$$B_6 = \Delta T_{a1}(A_4t + A_5)(1 - \tau_1A_1)(\tau_2A_1 - 1)$$

and

$$B_7 = A_1(\tau_2A_1 - 1)(\tau_1A_1 - 1)(e^{-t_{dl}/\tau_1} + e^{-t_{dl}/\tau_2} - e^{-t_{dl}/\tau_1 - t_{dl}/\tau_2} - 1).$$

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